

"بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ"

ELECTRIC TRACTION

Systems of Electric Traction.

Systems of electric traction may be divided into two main groups; in one group the vehicles receive their power from a distributing network fed by a few large generating stations, in the other the vehicles generate or carry their own energy. The first group may be further subdivided into systems operating with d.c., such as tramways, trolley-buses, and railways, and systems operating with a.c., viz. railways. The second group may be subdivided according to the nature of generation or storage; thus there are the diesel-electric trains and ships, the petrol-electric trucks and lorries, and the battery-driven road vehicles.

Advantages and Disadvantages of Electric Traction.

One of the most obvious advantages of electric traction, especially of the first group, is the cleanliness it possesses above all other systems. This alone makes it essential for use in the underground and tube railways.

Another well-known advantage is the rapid and smooth acceleration and braking possible with electric traction; an electric locomotive has an acceleration of 1.0 to 2.0 m.p.h. per sec., compared with the 0.4 to 0.5 of a steam locomotive. This is of special importance in suburban traffic, where very frequent trains must be run at early morning, mid-day, and evening, and where frequent stops occur. With a given track capacity, electric traction can carry up to 100 per cent

more people than steam traction, because of the higher average speed over short runs with frequent stops.

The size of stations in towns is limited very strictly by financial considerations, and the superior manoeuvrability of the electric locomotive enables twice as many to be used in a station of a given size.

An electric locomotive needs much less time for maintenance and repair than a steam locomotive, so that fewer are required for a given volume of traffic, also the cost of maintenance and repair per locomotive is less by about 50 per cent. It can be used immediately, whereas a steam locomotive takes about two hours to get up steam; this results in a better utilization of drivers' time.

Because of the absence of smoke and sparks, there is a greater safety in driving and an absence of damage to the buildings and apparatus due to the corrosive smoke fumes.

A saving is caused by the absence of coaling and water depots, and also the time of coaling.

The superior braking methods allow less wear on the brake shoes, and in some cases a saving of energy, which is returned to the supply instead of being wasted as heat in the brake shoes.

The main disadvantage is the capital outlay required to convert from steam to electric traction. It is certain that if this difficulty were overcome, the other disadvantages would not prevent a rapid conversion to electric traction.

Another disadvantage is that a failure of the power supply for a few minutes may cause a disorganization of the service for one or two hours. Increased reliability of supply will render failures very infrequent, and improved organization will diminish the time of interruption of service from each failure. It is known that a layer of ice on the conductor rails may prevent the train from collecting the power which is available; this trouble is easily overcome by running a service locomotive up and down the line to prevent the formation of ice.

Steam locomotives use their steam for heating the compartments very cheaply, whereas electric locomotives require to draw power for this purpose at a greater cost.

In many cases telephone and telegraph lines run along the track, and these will experience considerable interference from the power lines. Either the lines must be moved away from the track, or they must be replaced by cables, and a considerable expense—up to 15 per cent of the total cost—may be incurred.

As already stated, the main difficulty is the capital cost to change from steam to electric traction on the railways. By far the major part of the cost is in the overhead equipment and feeders, and this is avoided in the use of diesel-electric traction. In this system the locomotive carries diesel engines which drive a d.c. generator that supplies power to the motor.

The diesel engine is run at a constant speed so that its power output is always available, whilst the electric drive takes this power available at all speeds of the locomotive.

Diesel-electric locomotives have been made efficient and streamlined, so that very high speeds are available. The main disadvantage in an. country is that the oil fuel has to be imported; if the extraction of oil from coal becomes an economic process, it is probable that the main railway lines will be converted to diesel-electric traction.

Petrol-electric traction has been used so far in heavy lorries and buses. The advantage is that the electric conversion produces a very fine and continuous control; thus the lorry can move slowly at an imperceptible speed and yet it can creep up the steepest slope without throttling the engine.

Battery-driven vehicles are being quickly introduced, as they are found very useful as light delivery vans and platform trucks. They are easy to control and very convenient to use. As road vehicles they suffer from the disadvantage of having a limited range and speed.

Electrification Systems.

D.C. and a.c. systems are used, the latter being single-phase or three-phase.

For tramcars the supply is about 600 volts d.c., and the rails act as the return circuit. There are regulations relating to the return circuit in order to prevent damage due to leakage currents. The track is designed to have good electrical continuity and conductivity so that the return current does not spread out much. The track is connected to the negative pole of the supply system, and must be such that the potential difference between any two points on it is not greater than 7 volts. When the current is high, it is not practicable to limit the potential difference by having an enormous return rail, but instead use is made of negative boosting in a way that will be described later. When the return circuit is near pipes, the potential of the return must not be more than 1 volt above earth or more than 1 volt below earth potential. The supply is either underground in a conduit or overhead on a trolley wire. When a trolley wire is used, the voltage at the generating station must not exceed 650 volts and at the trolley wire 550 volts. The trolley wire must be divided into sections of not more than half a mile, between every two of which there must be emergency switches. When the track is run on private ground 1500 volts d.c. is favoured. For trolley buses the supply is at 600 volts, both lines being overhead and insulated from ground. As the return circuit is not earthed there is no fear of electrolysis, and negative feeder boosters are not required.

Single-phase main-line locomotives use 15 kV. at $16\frac{2}{3}$ cycles in Austria, Germany, Sweden, and Switzerland and other countries; in Pennsylvania single-phase of 11 kV. at 25 cycles is used.

Three-phase is used in some mountainous districts, e.g. Italy. The voltage is 3 600 volts between phases; two overhead conductors are used with the rail as the third phase. The necessity for two collectors is a disadvantage. No transformers are required as the induction motors run at the line voltage. The frequency of the supply is $16\frac{2}{3}$ cycles. Regeneration is automatic, and this is very useful in mountainous districts. The absence of commutators is a great advantage and lowers the cost of upkeep. As the induction motor is sensitive to speed variations, it is impossible to

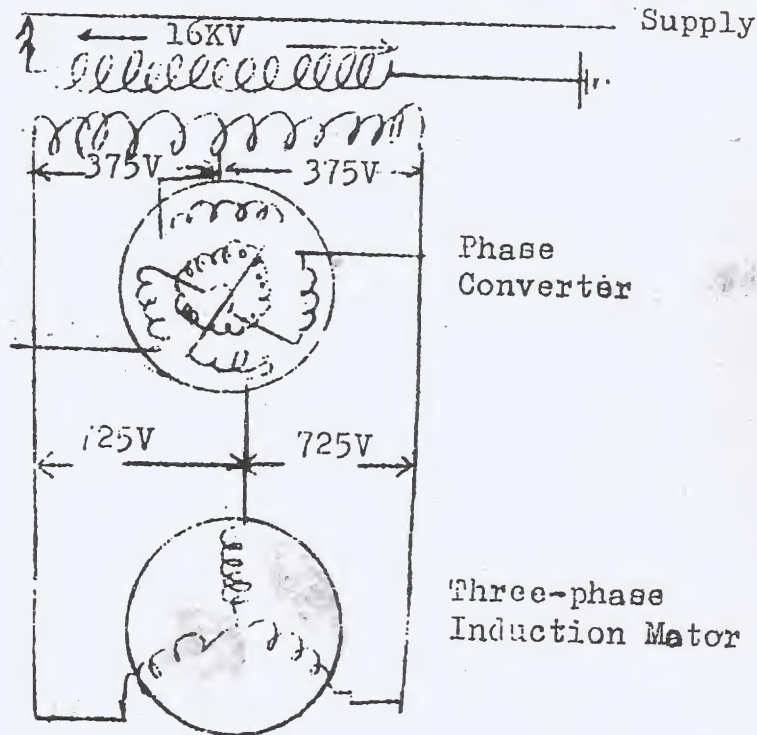


Fig. 1 Split-phase Traction System (Kando)

the multiple unit method; for the motors running on worn wheels would rotate faster than those on new wheels and would do little work, or even no work if the wheels were worn enough. A locomotive provides all the tractive effort.

Split-phase is used in the Kando system in which the supply is single-phase of 16 kV. at 50 cycles. A phase inverter supplies the motor, which is a three-phase induction motor (see Fig. 1).

The trend seems to be to install no more three-phase systems, but either high voltage d.c. or single-phase industrial frequency; the latter is likely to become a serious rival of the older systems.

Mechanics of Train Movement.

Fig 2 shows a diagram of the essential driving parts of an electric locomotive. The armature of the motor experiences a torque T (in lb. ft.), and it has attached to it a pinion of diameter p . There is a tractive effort F at the edge of the pinion, where $T = \frac{1}{2}pF$. This tractive effort is transferred to the driving wheel (diameter D) by means of the gear wheel (diameter d), so that tractive effort on the driving wheel is

$$F = \frac{3}{2} F(d/D) = \left. \begin{aligned} &T \times (2/p) \times (d/D) \\ &= T \times (2G/D) \end{aligned} \right\} \quad (1)$$

where η is the efficiency of the gear and G is the gear ratio d/p .

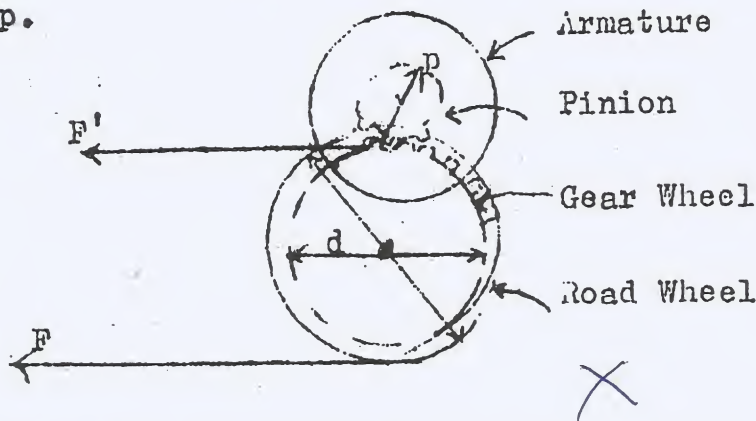


Fig.2 Driving Parts Of Electric Locomotive

The magnitude of the tractive effort that can be usefully employed depends upon the weight on the driving wheels and the adhesion of the driving wheels to the rails. The coefficient of adhesion is defined as

$$\eta_{ad} = \frac{\text{Tractive effort to slip the wheels}}{\text{Adhesive weight}}$$

and the following table gives values for electric tractors on dry rails.

Speed, m.p.h.	0	10	20	30	40	50
Coefficient of adhesion	0.25	0.18	0.14	0.12	0.10	0.09

If the rails are greasy the value may be as low as 0.08.

A very important advantage of electric traction is that in a

motor coach 100 per cent of the weight is on the driving wheels, in an electric locomotive 70 per cent or more, but in a steam passenger locomotive less than 50 per cent. Moreover, the coefficient of adhesion in electric traction is greater than in steam traction; this is because (i) the torque in electric traction is continuous while in steam traction it is pulsating and the uneven torque sets up a jolting and skidding, and (ii) in electric traction the driving wheels are distributed along the length of the train, whilst in steam traction they are close together. Thus the maximum possible tractive effort is much greater in electric traction than in steam traction.

The maximum possible acceleration can be found from the coefficient of adhesion. Suppose that the whole of the weight is on the driving wheels and the locomotive is running also; then the maximum tractive effort is 0.25 times its weight, the acceleration is 0.25 times g , viz.

$$\begin{aligned} 0.25 \times 32.2 &= 8.1 \text{ ft. per sec. per sec.} \\ &= 5.5 \text{ m.p.h. per sec.} \end{aligned}$$

If the weight of the motor coaches is only one-third of the total weight of the train, the acceleration cannot exceed one-third this value, viz. 1.8 m.p.h. per sec. This is the limit of value that is obtained in practice. Braking retardation can be much greater than the acceleration, as the brakes act on all wheels: values of 3.2 m.p.h. per sec. can be obtained.

If a tractive effort of F_a lb. wt. acts on a mass of W tons, the acceleration is

$$\begin{aligned} &= (F_a \times 32.2) / 2 \times 240W \text{ ft. per sec. per sec.} \\ &= F_a / 102W \text{ m.p.h. per sec.} \end{aligned} \quad (2)$$

When the train accelerates, kinetic energy is produced in two ways, by the linear motion of the train, and by the rotation of the wheels and motors: the former is $\frac{1}{2}Wv^2$, where v is the velocity of the train, and the latter is

$$\sum \frac{1}{2} I \omega^2 = \sum I (\frac{1}{2} v^2 / r^2) = \frac{1}{2} m v^2,$$

where $m = \sum (I/r^2)$; I being the moment of inertia of a rotating part and W its angular velocity.

The sigma is taken for all rotating parts. This means that the effective value of the mass of the train is $W + m$; in practice m is from 8 to 15 per cent of the dead weight W . Equation (2) then becomes

$$\alpha = F_a / 102(W + m) \text{ m.p.h. per sec.} \quad (3a)$$

or

$$F_a = 102 \alpha (W + m) = 102 \alpha W_e, \quad (4)$$

where $W_e = W + m$; and is called the effective or accelerating mass of the train. The tractive effort F_a is that required for acceleration; in practice the total tractive effort supplied by the motors must be equal to this plus the effort to overcome the train resistance, and gravitation if the train is on a slope. The tractive effort to overcome train resistance is

$$F_r = Wr,$$

where r = specific train resistance, and is a function of the velocity for a given train. The tractive effort to overcome

gravity on a slope of percentage gradient G is

$$\begin{aligned} F &= \pm WG / 100 \text{ tons} \\ &= \pm 22.4WG \text{ lb. Wt.}, \end{aligned}$$

where the positive sign is used for an up-gradient and the negative for a down-gradient. The total tractive effort is

$$\begin{aligned} F_t &= F_a + F_r + F_g \\ &= (102 \alpha W_e + W_r \pm 22.4WG) \text{ lb. wt.} \end{aligned} \quad (5)$$

The power output of the driving axles is

$$P = F_t v \text{ ft. lb. wt. per sec.},$$

where v is in ft. per sec., so that

$$P = F_t V \times \frac{5280 \times 0.746}{60 \times 33000} \text{ kW.}$$

$$= 0.00199 F_t V \text{ kW.},$$

$$\approx 0.002 F_t V \text{ kW.}$$

where V is in m.p.h.

Example.

A motor-coach train weighing 200 tons is accelerated up a gradient of 1 in 200 at a mean acceleration of 1.2 m.p.h.p.s. up to a speed of 30 m.p.h. Find (1) the tractive effort required, and (2) the output at the end of the accelerating period. The train resistance is 10 lb. per ton and the effective weight is 10% more than the dead weight.

In this case $\alpha = 1.2$, $W = 200$, and $m = 0.1 \times 200 = 20$, so that $W_e = 220$, $r = 10$, and $G = \frac{1}{2} (1 \text{ in } 200)$.

By equation (5) the required tractive effort is

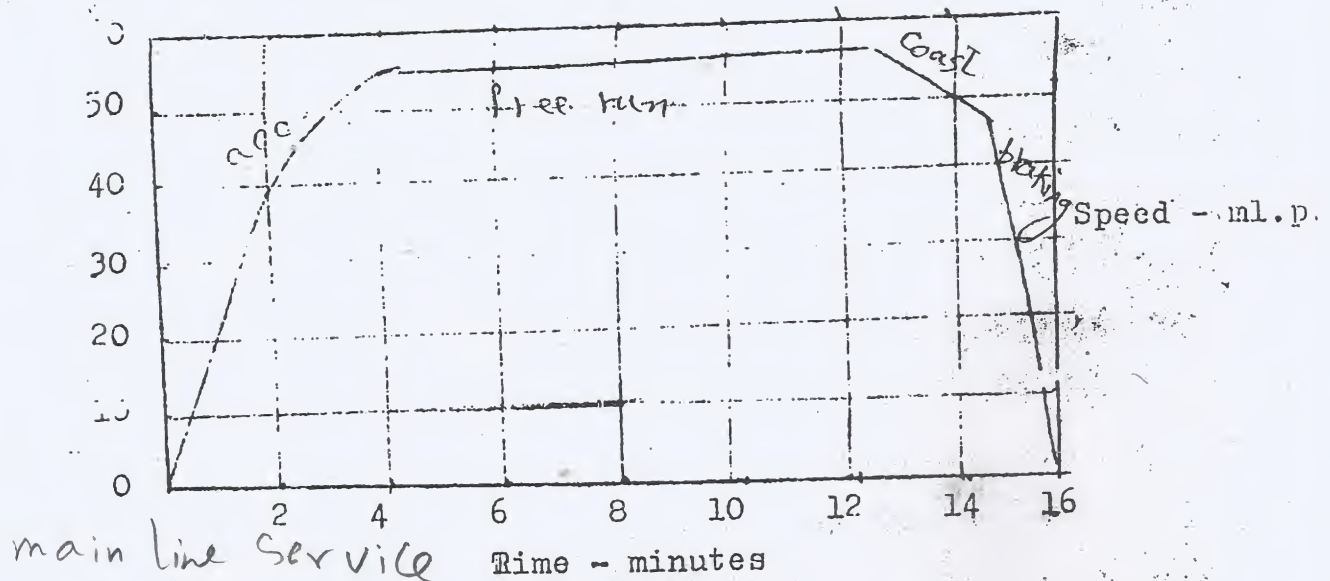
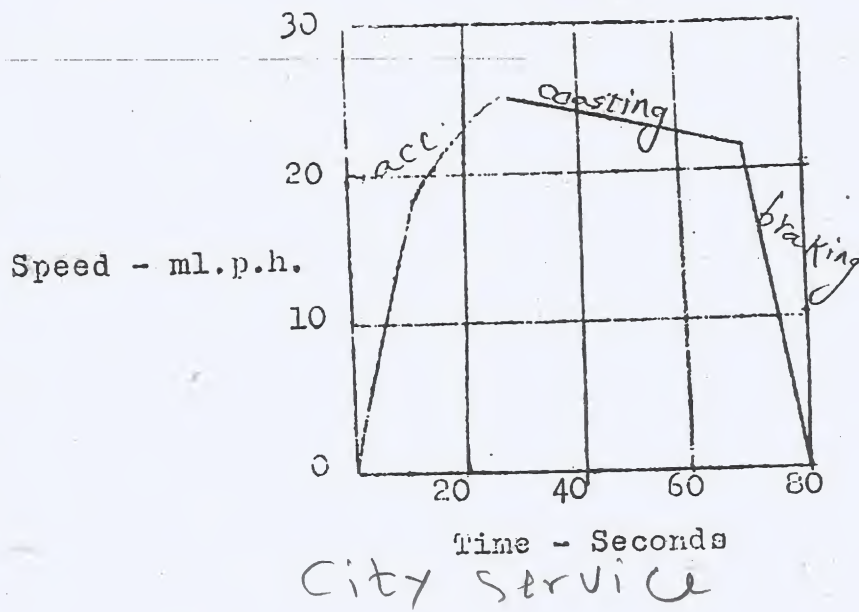
$$\begin{aligned} F_t &= 102 \times 1.2 \times 220 + 200 \times 10 + 22.4 \times 200 \times \frac{1}{2} \\ &= 27\,000 + 2\,000 + 2\,240 \\ &= \underline{\underline{31\,240\text{ lb. wt.}}} \end{aligned}$$

At the end of the accelerating period $V = 30$, so that the power is

$$\begin{aligned} P &= 0.00199 \times 31\,240 \times 30 \\ &= \underline{\underline{1\,870\text{ k W.}}} \end{aligned}$$

Speed-time Curves.

If a curve is plotted with time (in seconds or minutes) as the abscissa and the speed (in miles per hour) as the ordinate, the complete information of the motion of the train is represented. The acceleration at any instant or speed is found by drawing a tangent at the corresponding point on the curve and calculating the slope of this tangent; the acceleration is given usually in miles per hour per second (1 m.p.h.p.s. is equal to 1.47 ft. per sec. per sec). The distance covered in a given time is represented by the area between the curve, the time axis, and the ordinates through the instants between which the time is taken. Fig. 2 shows the speed-time curves for city and main-line services.



Speed-Time Curves: City Service, Main-Line Service

The initial acceleration in the city service is seen to be 10 m.p.h. per 8.3 sec. = 1.21 m.p.h.p.s. The total distance between stops in the main-line service is represented by the area of 37.5 squares, each of which corresponds to a distance of

$$10 \text{ m.p.h.} \times 2 \text{ min.} = \frac{10 \times 2}{60} \text{ miles} = \frac{1}{3} \text{ mile,}$$

so that the total distance is $37.5 \div 3 = 12.5$ miles.

$$\text{average speed} = \frac{\text{Distance covered}}{\text{Time of run}} = \text{m.p.h.}$$

There are usually four periods in the run, viz. acceleration; constant speed or free running; coasting, when the power is shut off and the train slows down gradually because of resistances to motion; and braking. In the speed-time curve show, for a main-line service these periods are 6, 7, $1\frac{3}{4}$ and $1\frac{1}{4}$ min. respectively. In short runs, such as the city and suburban services, the free running period may not exist. The acceleration period consists of two parts. In the first part the motor tractive effort is kept constant by means of resistance notching, or more recently by the metadyne; this occurs until all the resistances are switched out and a speed V_1 is reached (see Fig. 3). The tractive effort available for acceleration, and climbing if necessary, is F , where F is the difference of the motor tractive effort and the train resistance. The acceleration in this period is nearly constant. In the second part of the acceleration period the motor tractive effort is the maximum that the motor can give at the speed,

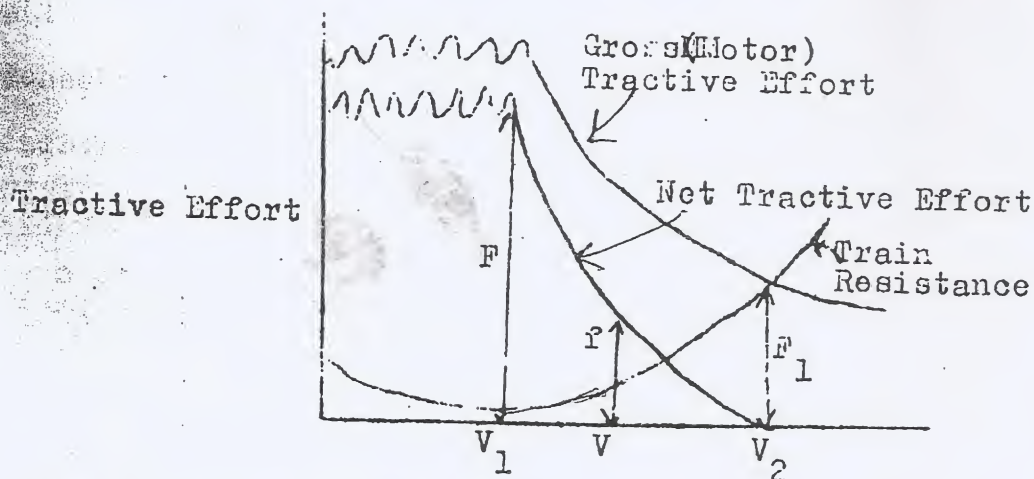


Fig.3 Tractive Effort Versus Speed .

Falls rapidly with the speed. The train resistance, however, increases, slowly to begin with and then rapidly, until at a certain speed V_2 the motor tractive effort is equal to the train resistance. At any speed between V_1 and V_2 the tractive effort available for acceleration (and climbing) is f , which decreases from F at V_1 to zero at V_2 . The acceleration between these speeds therefore decreases from the maximum value (about 1 or 2 m.p.h.p.s.) to zero. V_2 is the maximum possible speed, and requires a motor tractive effort F_1 to maintain it. If the power is shut off, the train resistance slows the train; at a speed V the decelerating force is due to the train resistance at that speed. If the curves of motor tractive effort and train resistance versus speed are known, the foregoing method enables the acceleration and deceleration of the train at any speed to be found. It will be shown later how the speed-time and distance-time curves of the train can be calculated from the acceleration—or deceleration—speed curves.

There are three speeds of importance: the crest speed, which is the maximum speed attained on the run; the average speed, which is the mean speed from start to stop; and the schedule speed, which is the mean speed when the stop period is included. Thus in the speed-time curve shown for a main-line service the crest speed is 56 m.p.h., and the average speed is

$$(12.5 \times 60)/16 = 46.8 \text{ m.p.h.}$$

If the stops are 2 min., the schedule speed is

$$(12.5 \times 60) / (16 + 2) = 41.6 \text{ m.p.h.}$$

Simplified Speed-Time Curves.

The speed-time curve of a city service can be replaced by a quadrilateral (Fig. 4) (a) or a trapezoid (Fig. 4 (b)), whilst that of a main-line service is best and most easily replaced by a trapezoid (Fig. 4 (c)). It is much easier to calculate the performance of the train from the simplified speed-time curves, and the results are accurate enough for most practical

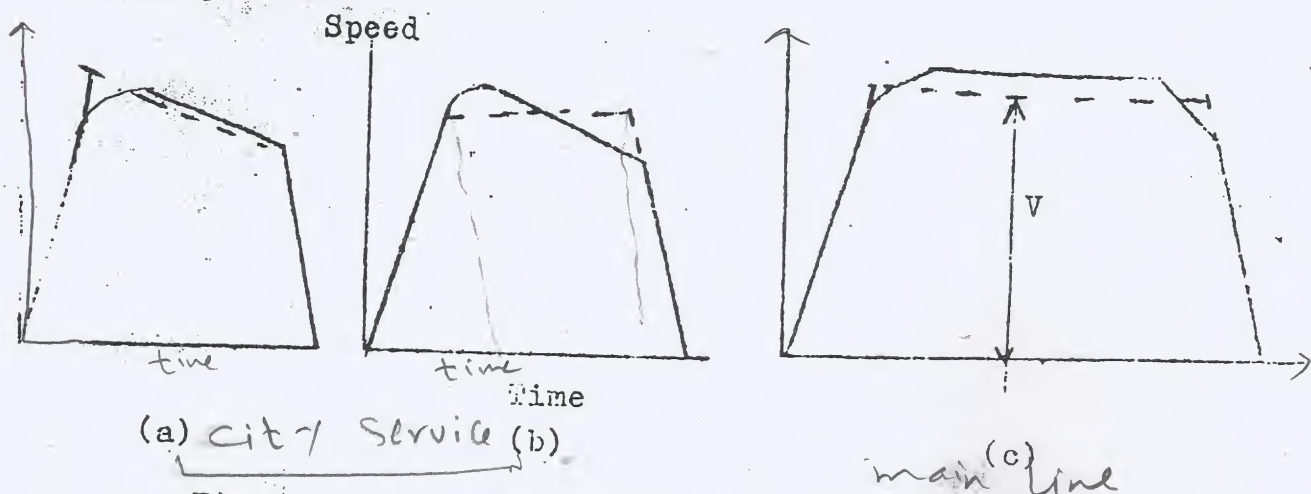


Fig.4 Approximate Speed-Time Curves

purposes. The following examples illustrate the method of calculation.

Example.

The time-speed diagram of an electric train is represented by a uniform acceleration of a m.p.h.p.s., coasting speed of V m.p.h., and uniform braking retardation of b m.p.h.p.s.

the time taken to run a distance of S miles between stops is T sec., show that

$$V = \frac{1}{2k} [T - \sqrt{T^2 - 14400Sk}], \text{ where } k = (a + b)/2ab.$$

In this case it is assumed that the coasting speed is constant—it would be better to call this free running—so that the speed-time curve is as shown in Fig. 4 (c). The acceleration is a and the final speed V, so that the duration of acceleration is V/a and the distance travelled in this time is

$$\frac{1}{2} a (V/a)^2 = \frac{1}{2} (V^2/a).$$

Similarly the duration of braking is V/b and the distance travelled in this time is $V^2/2b$. The time of free running is thus

$$T - V/a - V/b,$$

and the distance travelled in this time

$$V(T - V/a - V/b).$$

The total distance is thus

$$\begin{aligned} S &= V^2/2a + V^2/2b + V(T - V/a - V/b) \\ &= VT - (V^2/2a + V^2/2b). \end{aligned}$$

This is a quadratic equation for V, viz.

$$V^2(1/2a + 1/2b) - VT + S = 0.$$

In this equation a and b are in m.p.h.p.s., V in m.p.h., T in sec., so that S is in

$$\text{miles per hour} \times \text{seconds} = \frac{1}{3600} \text{ miles.}$$

If we want the distance in miles, S say, we have

$$S/3600 = S \text{ or } S' = 3600 S.$$

The equation for V becomes

$$kV^2 - VT + 3600S = 0, \quad \dots\dots\dots (7)$$

where

$$k = 1/2a + 1/2b = (a + b)/2ab$$

The solution is

$$V = T/2k \pm (1/2k) \sqrt{T^2 - 4 \times 3600kS}$$

$$= (1/2k) (T \pm \sqrt{(T^2 - 14400Sk)}).$$

To determine the correct sign we note that the time of free running is

$$T - V/a - V/b = T - 2kV = \pm \sqrt{(T^2 - 14400Sk)}.$$

It is thus necessary to take the lower sign, and we have

$$V = (1/2k) [T - \sqrt{(T^2 - 14400Sk)}]$$

and the time of free running is $\sqrt{(T^2 - 14400Sk)}$

Effect on Schedule Speed of Acceleration, Braking and Distance.

Equation (7) gives a general relation, for the trapezoidal speedtime curve, between the maximum speed, acceleration, braking retardation, distance, and time of running. Its main use is for finding the maximum speed necessary or the acceleration required for a desired schedule speed on a given line. The following example shows how this is done.

Example.

An electric train operating on a suburban service has a maximum running speed of 38 m.p.h. The average distance between stops is 2 200 yd. and the schedule speed including a station stop of 20 sec. is 25 m.p.h. Calculate the necessary acceleration, allowing a maximum ^{braking} retardation of 2.5 m.p.h.p.s. (b)

As the distance S is 2 200 yd. = 1.25 miles and the schedule speed is 25 m.p.h., the time of travel plus the stop of 20 sec. is $1.25/25 = 0.05$ hr., i.e. 3 min. or 180 sec. The time of travel, T, is thus $180 - 20 = 160$ sec. The maximum speed V is 38 m.p.h. Equation (7) can be written as

$$k = (VT - 3600S)/V^2 = 1/2a + 1/2b.$$

so that

$$1/a = 2(VT - 3600S)/V^2 - 1/b$$

$$= \frac{2(38 \times 160 - 3600 \times 1.25)}{38^2} - \frac{1}{2.5}$$

$$= 1.76$$

and the required acceleration is

$$a = 1/1.76 = \underline{0.57} \text{ m.p.h.p.s.}$$

Calculation Of Speed-Time Curve

We have seen that the tractive effort available for acceleration is the total tractive effort less that required to overcome train resistance and gravity. We can rewrite equation (5) as

$$F_a = F_t - F_r - F_g$$

or

$$102 W_a^{\text{ton}} = F_t - W_r - 22.4WG,$$

that the acceleration is

$$\alpha = (F_t - W_r - 22.4WG) / 102W_e \quad (5)$$

The total tractive effort and the train resistance are given, in the form of curves, as functions of the speed, as shown in Fig. (3). We can then calculate the acceleration at any speed v . As

$$\alpha = dv/dt, \quad dt = dv/\alpha$$

and thus
$$t = \int \frac{1}{\alpha} dv \quad (6)$$

Equation (8) expresses time as a function of the speed, i.e. it gives the time to attain a certain speed under the varying acceleration. By making t the abscissa and v the ordinate we obtain the speed-time curve, from which the complete performance is easily found in the way shown above.

During coasting and braking both α and dv are negative, but the method is just the same.

If α is a simple function of v the integration may be possible in known functions; otherwise a graphical method must be used. The method described is applicable to rotating machinery as well as to traction, in which case α is the angular acceleration, v is the angular velocity, and we replace mass by the moment of inertia.

The graphical method of obtaining the speed-time curve is the following. We plot $1/\alpha$ against v ; Fig (5) shows this curve for the traction system represented by the curves of Fig. (3). $1/\alpha$ is approximately constant up to the speed v_1 ,

increases to ∞ at speed V_2 . The time t to reach a speed V is given by the shaded area shown in Fig.5. This is done for several values of V , and a table of t against V is written down from which the speed-time curve is plotted.

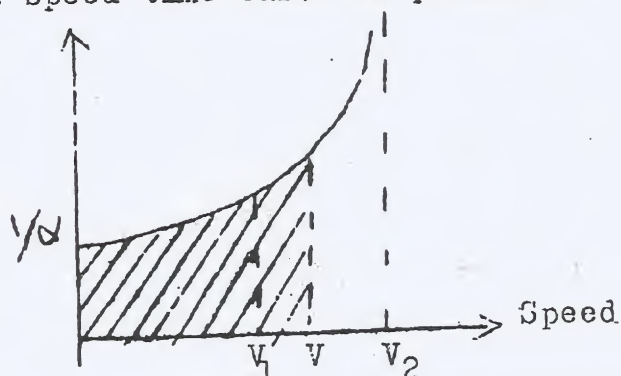


Fig.5

In practice the current-speed and current-tractive effort curves of the motors are given, and these with the condition of maximum current give the tractive effort-speed curve of the motors.

EXAMPLE. A train has a total weight of 116 tons and is equipped with four motors each of 275 h.p. The characteristics of the motor and the train resistance are given by the following table

Current (A.)	100	200	300	400	500
Speed (m.p.h.)	51	31.4	26.4	23.9	22.1
Tractive effort (lb. wt.)	390	1 600	2 960	4 330	5 690
Train resistance (lb.wt.per ton)	1.1	10	9	9	10

The ratio of the effective weight to the dead weight of the train is 1.1 to 1. The mean accelerating current is 425 A. per motor, and the braking retardation is $\frac{1}{2}$ m.p.h.p.s. A run of 0.86 mile is to be made in 115 sec., there being an average up grade of 0.119 per cent. Calculate the r.m.s. current per motor for the run assuming the train resistance during coasting equals 10 lb.wt./ton

Fig.6 shows the current-speed and current-tractive effort curves per motor. The current is not allowed to exceed 425 A. by notching, so that the maximum tractive effort per motor is

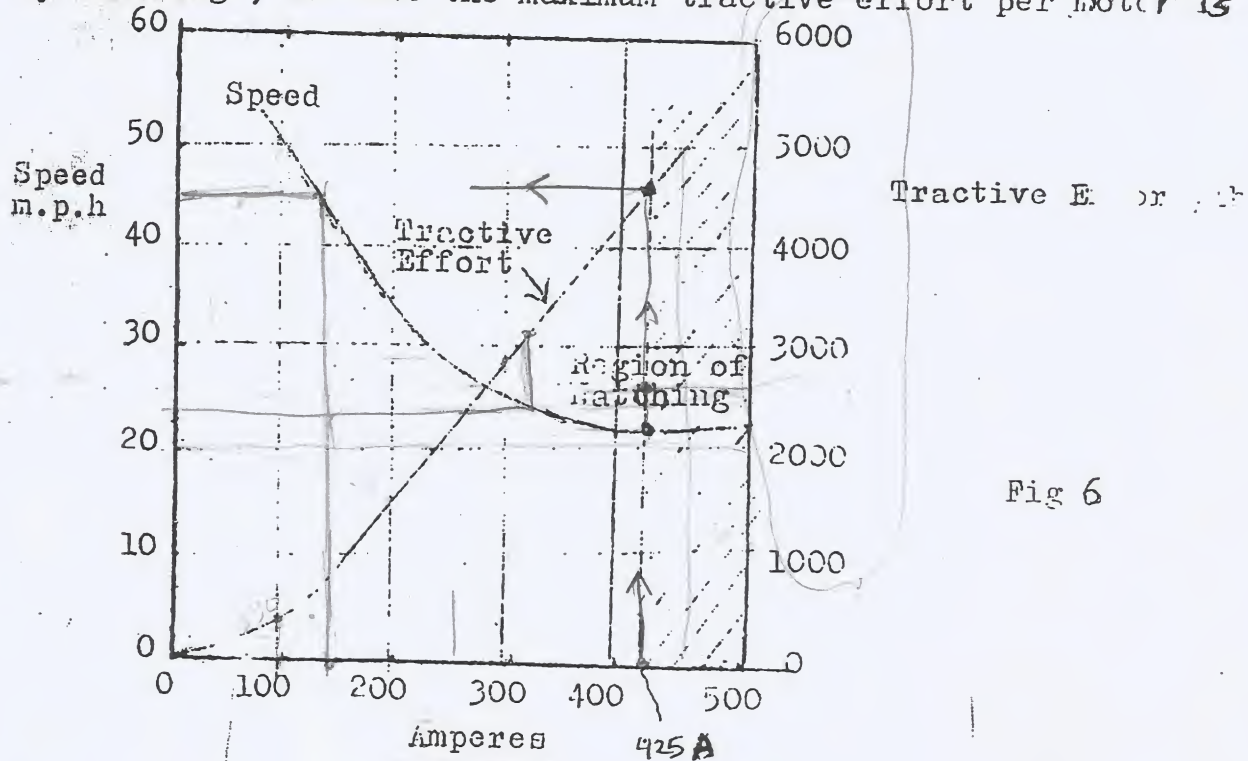


Fig 6

A 650, lb. wt. and this acts until the speed reaches 23.5 m.p.h. The tractive effort required to overcome gravity is $22.4 \times 116 \times 0.119 = 310$ lb. wt. We now construct a table of F_t , F_r , F_g and F_a against speed, and from F_a we calculate the acceleration a from equation (4), viz.

$$a = \frac{F_a}{102W_e} = \frac{F_a}{102 \times 1.1 \times 116} = \frac{F_a}{13000}$$

since W_e is 1.1 x the dead mass

Speed .	23.5	26	28	30	35	40	45	X
F_t .	18 600	12 800	9 760	7 600	4 800	3 200	2 080	
P_r .	1 160	1 050	1 050	1 160	1 350	1 580	1 860	
P_g .				310				
P_a .	17130	11440	8400	6130	3140	1310	- 90	
a .	1.32	0.88	0.65	0.47	0.24	0.10	-0.007	
$1/a$.	0.76	1.14	1.54	2.13	4.16	10.0		X

Fig.7 shows the curve of $1/a$ against speed. Unit length along the abscissa is 1 m.p.h. and unit length along the ordinate is $1/10$ m.p.h.p.s., so that a unit square represents 0.1 sec. Counting the squares between ordinates we get the time between given speeds; thus the time from 0 to 23.5 m.p.h. is 17.3 sec., from 23.5 to 30 m.p.h. 8.4 sec., and so on. The speed-time curve is then the following.

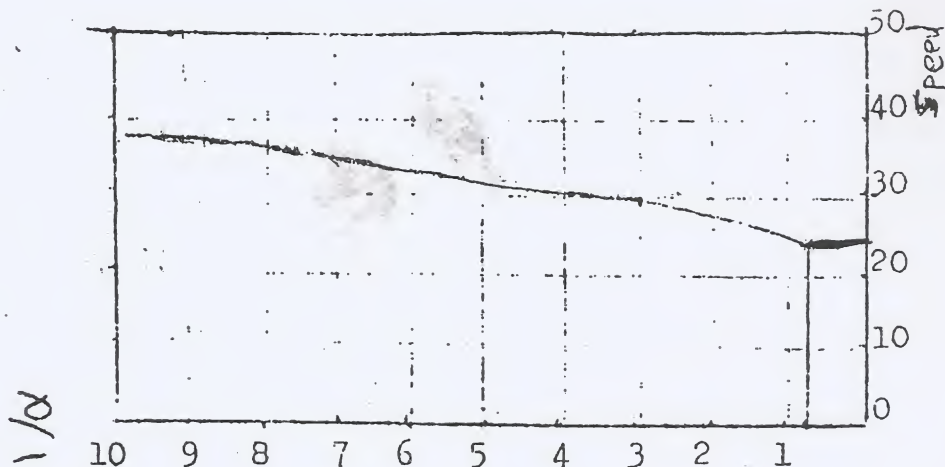
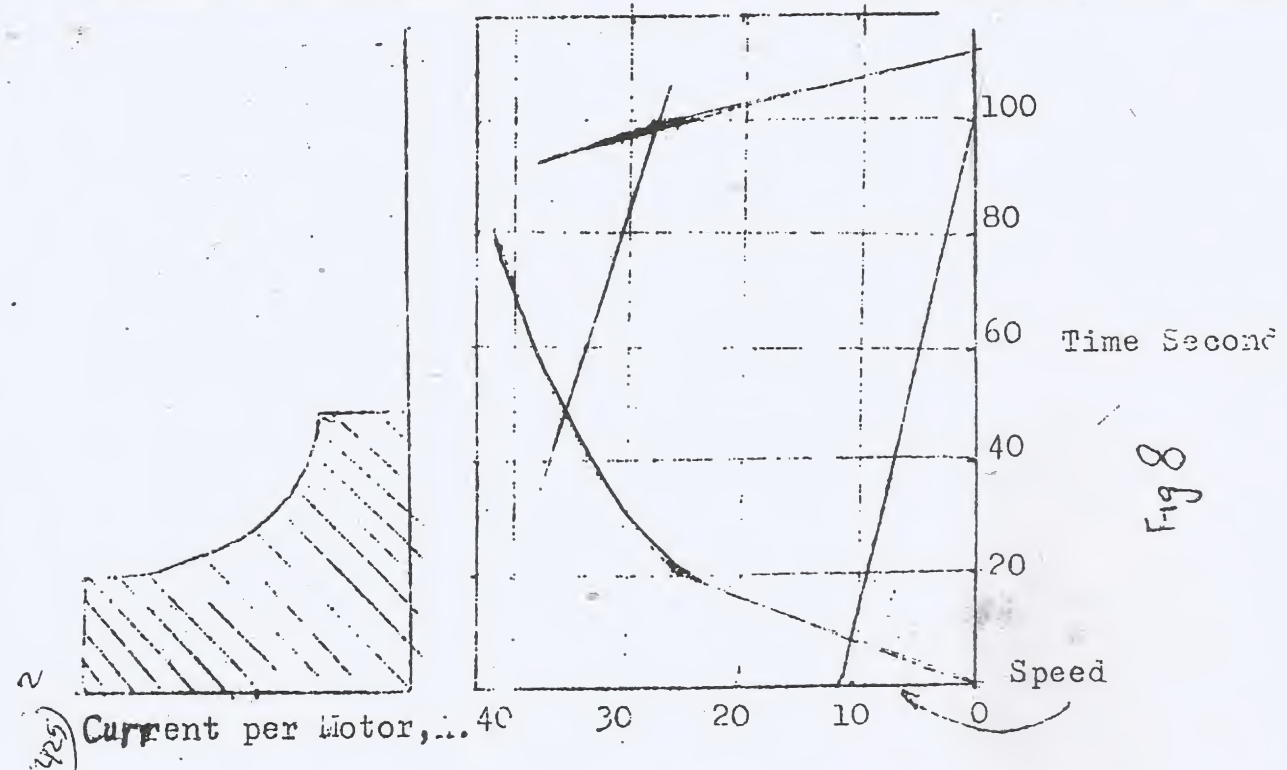
Speed	0	23.5	25	30	35	40
Time	0	17.9	18.5	25.7	40.7	86.2

Fig.8 shows the speed-time curve. The time of the run is 115 sec. and the braking retardation is 2 m.p.h.p.s., so that the speed-time curve for the braking period is a straight line through the point $t = 115$ on the time-axis with a slope of 2 m.p.h.p.s.; thus it goes through the point corresponding to $t = 100$ and $v = (115 - 100) \times 2 = 30$ m.p.h. As the run is

short there will be no free funning but a coasting period, during which there will be a retarding force due to train resistance and gravity. Let us take these as 1 160 and 31.0 respectively, and then the retardation during coasting is

$$1\ 470/13\ 000 = 0.113\ \text{m.p.h.p.s.}$$

The coasting period is represented by a line of this slope. The position of this line is such that the area under the composite curve, consisting of acceleration, coasting and braking curves, corresponds to the distance travelled, viz. 0.36 mile.



In the figure drawn, unit abscissa is 2 sec. and unit ordinate 1 m.p.h., so that unit square represents

$$1 \times \frac{2}{3600} \text{ miles} = \frac{1}{1800} \text{ miles}.$$

The number of unit squares under the composite curve must thus be $1800 \times 0.86 = 1550$.

A quick and easy way to find the position of the coasting line is to add up the squares contained between the acceleration and braking curves, the time axis and parallels to the latter.

Up to the parallel through a speed of 10 m.p.h. the number of squares is (540), up to 20 m.p.h. (1020), ^(25 m.p.h.) 30 m.p.h. (1605). The coasting line must lie therefore between the horizontal lines representing 30 and 35 m.p.h., rather nearer the higher line.

It is simple to move a ruler so as to be parallel to the coasting line and count the squares between the ruler and the line.

$V = 35$ m.p.h.: the position required is where the area is $1605 - 1550$ squares, and is shown in Fig.8

The run is made as follows. Acceleration for 45 sec. then a speed of 36 m.p.h. is reached, coasting for 55 sec. during which the speed drops to 29.5 m.p.h., and braking for the last 15 sec. The upper part of Fig.8 shows the current-time curve, which is derived from the speed-time curve and the current-speed curve of Fig.6

The r.m.s. current gives an indication of the beating of the motors. To find the r.m.s. we plot (current)² against time, and the area under this curve, divide by the total time of 1.15 sec., and take the square root. It is found that mean square

is 38 400 (amperes)² and the r.m.s. is 196 A.

Traction Motors. The types used are the series or compound motor for d.c., series for single-phase, and induction motor for threephase. The conditions of service are very severe, so that the traction motor is built on very robust lines. As it has to be protected against water and mud, it is totally enclosed, and if necessary ventilating ducts are specially arranged in the design.

The series and lightly compounded motors have a torque-speed (or current-speed) characteristic that shows a rapid variation of speed with torque, whilst a shunt (or an induction) motor has small variation of speed with torque. Fig.9 shows the two types of characteristic. Suppose we have two identical driving different wheels that are not connected by a rod : if one wheel is smaller, it has a larger angular velocity and the motor driving it will have a higher speed than the other. We represent the speeds by N'_0 and N''_0 , on either side of a normal speed N_0 . If the motors have a series characteristic, the torques are T'_0 and T''_0 , which are seen to be nearly equal, so that the motors share the load fairly. If the motors have a shunt characteristic, it is seen that they share the load very unequally; as shown in the figure, the higher speed motor is acting as a generator and is doing less than no useful work, whilst the other is doing more than the total mechanically necessary work. Whilst this would hardly occur in practice, it is nevertheless true that the motor would share the work very unequally, and for this reason motors with a shunt-characteristic

cannot be used on individual drive. Induction motors on ~~three~~-phase systems have their driving wheels linked by connecting rods.

Starting and Speed Control of D.C. Motors. The speed of a d.c. series motor can be varied either by varying the field or the voltage applied to it. The field can be varied by tapping on the field or by a shunt across it; the latter is called "field weakening." The voltage applied can be varied in three main ways, by means of series resistance or the series-parallel method, by the metadyne, by the Ward-Leonard system; the last method is not used in traction.

For the purpose of starting field control is clearly useless, and recourse is had either to series resistance (notching); the series-parallel method in conjunction with series resistance, or the metadyne.

Notching. The current in a series motor is given by

$$E - kNI = IR,$$

or

$$I = E/(R + kN),$$

where E is the voltage, N the speed, k a constant of the field and armature, and R the d.c. resistance of the armature and external resistance. At zero speed the current is E/R ; external resistance is inserted to limit I to some predetermined value, and as the motor speeds up the current drops the external resistance can be reduced eventually to zero. The current (or torque curve) is like that shown in

is

Fig.9. The method of calculating the steps of resistance is given in Electrical Technology, by H. Cotton (Sixth Edition, pages 143-6). In this method of starting, known as notching, the resistance is put in series with the motor so

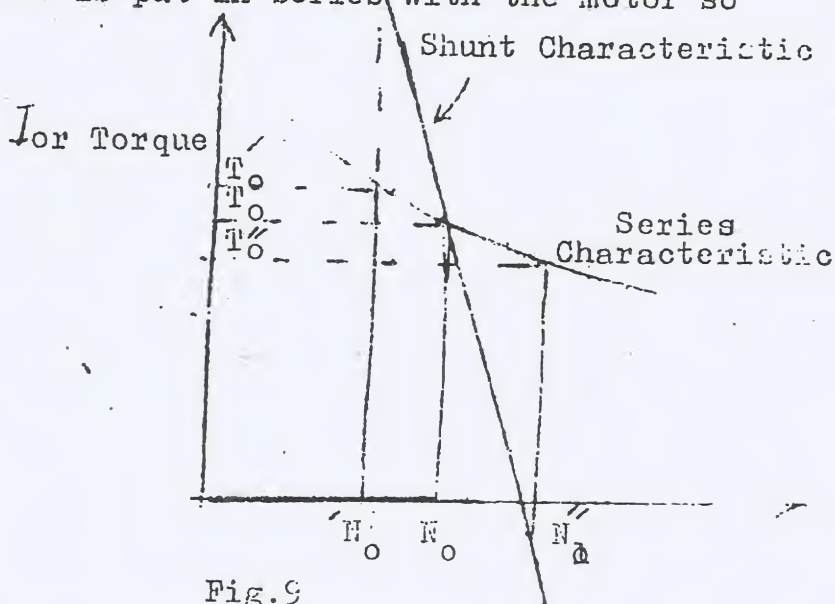


Fig.9

that the current has a certain maximum value, I_1 say, and remains until the speeding up reduces the current to a certain minimum value, I_2 say. The resistance is then reduced so that the current regains the value I_1 , and so on until no resistance is left.

Series-parallel Control. The current in the series resistance of the last method of starting a series d.c. motor involves a great waste. Part of this waste can be avoided by the seriesparallel method, when there are two or more motors.

If there are two motors, they can be started in series with a limiting resistance, run up to half speed (when the series resistance is zero and their total back e.m.f. is equal to the supply voltage), switched over into parallel with limiting resistance again, and then run up to full speed.

when the back e.m.f. of each is equal to the supply voltage.

If there are four motors, more combinations are possible, i.e. series, series-parallel, and parallel.

There are two main methods of effecting the change from series to parallel, the shunt-transition and the bridge-transition methods, which are shown in Figs. 10 A. and B. In the former method the motors are run up to the full series position, when the series resistance is cut down to zero. Then some series resistance is reinserted, and one motor is short-circuited. Then this motor has one end opened, and this end is connected across so that the motors are in parallel. The series resistance is then cut out as the motors speed up. There is a jerk in this system as one motor is shorted and ceases to act, and then another jerk when it is reinserted.

In the bridge-transition method, a resistance is put across each motor after the full series position is reached, and then shorting bar between the motors is removed, leaving the motors (each in series with a resistance) in parallel with each other. If the resistance across the motors have the correct value, the shorting bar has no current, since the arrangement is that of a Wheatstone bridge, and the transition is perfectly smooth.

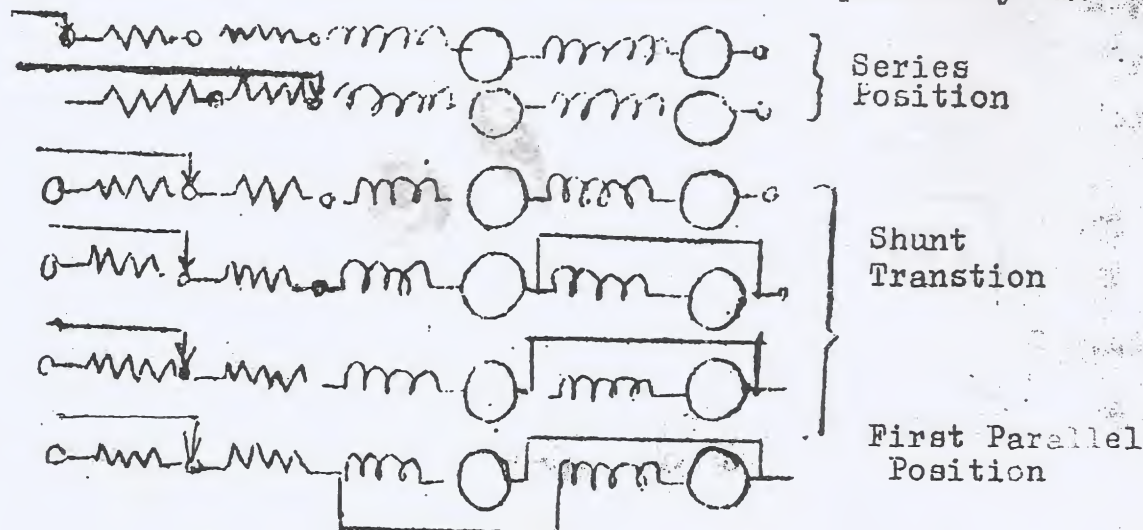


Fig. 10 Shunt-Transition Method

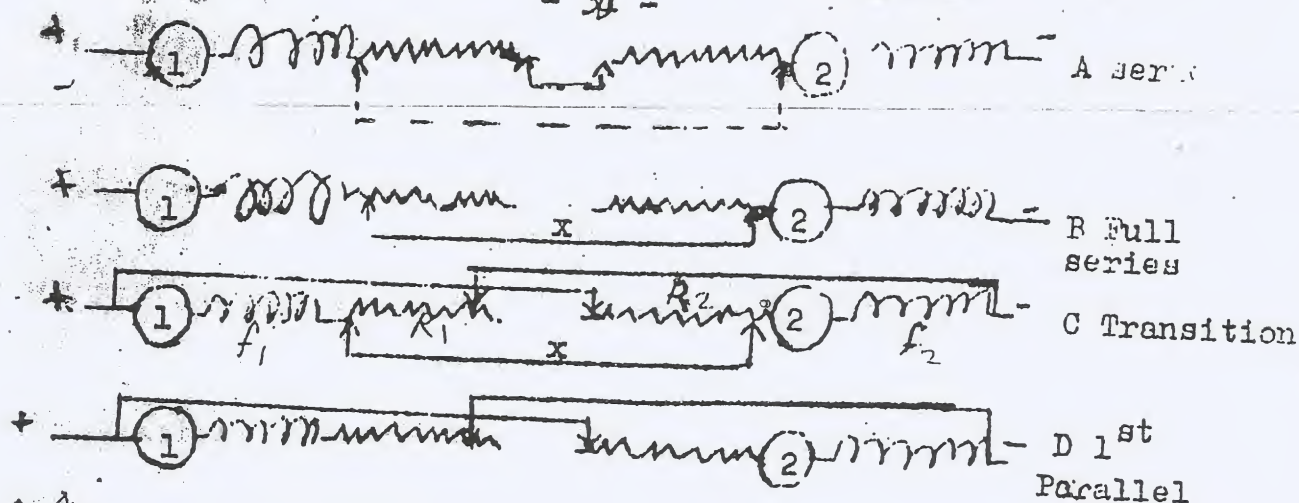


Fig. 10 B : Bridge-Transition Method

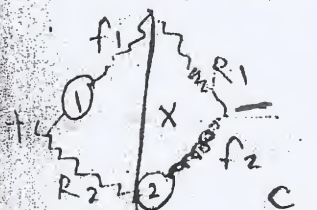


Fig. 11 shows a simplified form of the power diagram of a 1500 volt d.c. train equipment. L_1 , L_2 , L_3 and L_4 are line switches and are used in series pairs. Motors 1 and 2 are in series, and have three breaks L_3 , L_4 , and P; Motors 3 and 4 are in series and have the breaks L_1 , L_2 and M. Bridging contactors S_1 and S_2 have full line voltage of 1500 volts across them when the motors are in parallel, and for this reason there are two breaks. Resistance W protects the system at switch-on in case there is a fault in motor 1.

On the first notch, L_1 , L_2 and S close, and then L_3 and L_4 . The motors are then in series with full limiting resistance. Contactors R then operate on the following notches, and the motors are running in series on the full line voltage. Then S_1 and S_2 close and S opens, and the motor are still in series. Contactors R then open, M and

(1)

(2)

Close, S_1 and S_2 open, and the motors are then in parallel with full limiting resistance. Contactors R finally cut out this resistance and the final parallel arrangement is reached.

ENERGY SAVED BY SERIES-PARALLEL CONTROL; Let us consider the cases (i) where the motors are started in parallel with limiting resistance, and (ii) where they are started by the series-parallel method. In both cases we assume that the series limiting resistances are continuously varied so that the current through each motor, whether in series or parallel, is equal to the maximum permissible value. It follows that the motor produces a constant torque, in whatever combination it finds itself, and thus there is the same constant acceleration in both methods. We will assume further that armature and series field resistances can be ignored, as they are small compared with the limiting resistances.

Fig. 12 (a) shows the electrical conditions in the first case. The total current drawn from the supply is $2I$, where I is the maximum permissible value per motor. The speed, and with it the back e.m.f., increases with time, until the

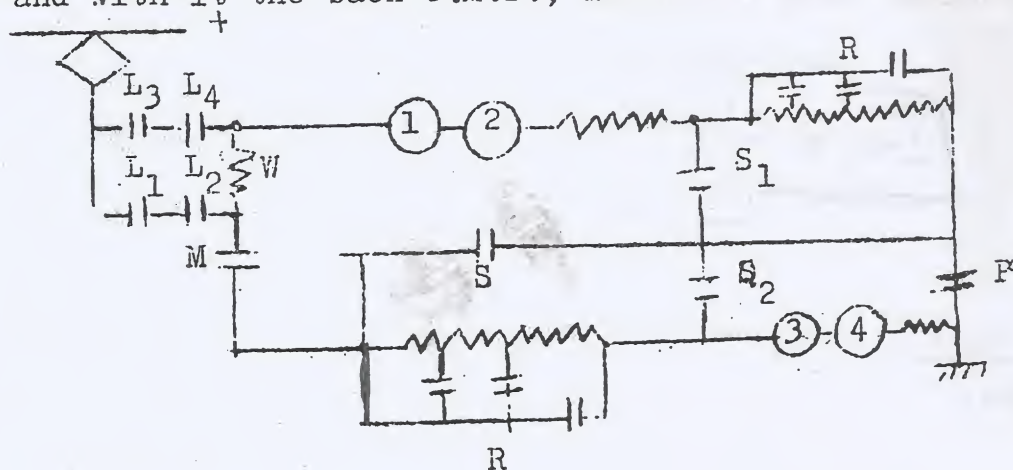
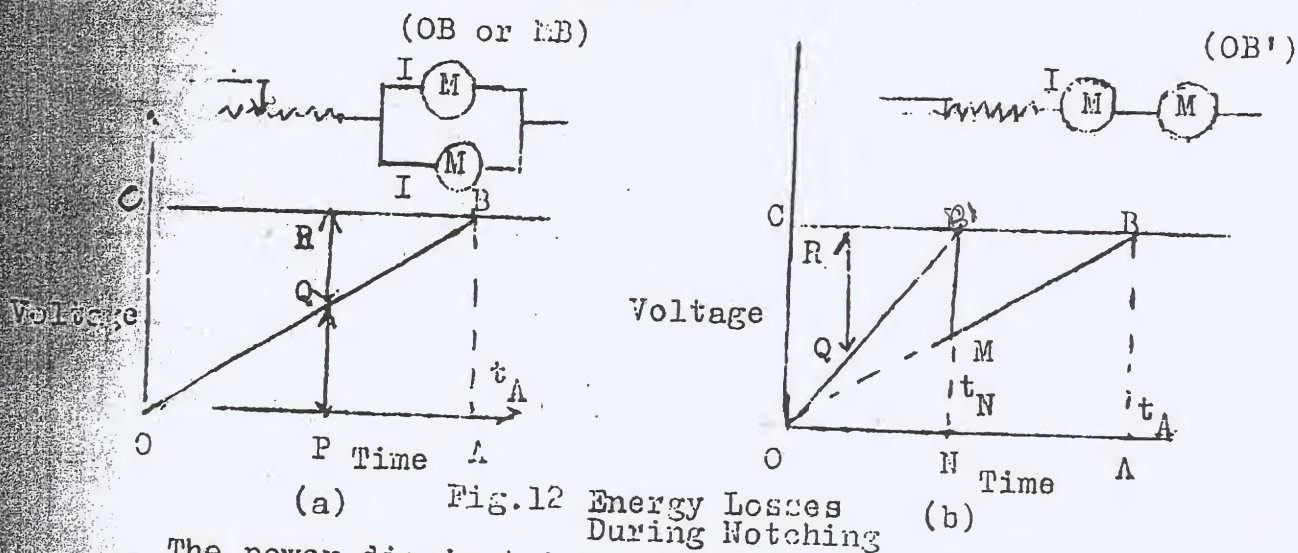


Fig. 11 Power Diagram Of 1500 V. D.C. Train Equipment

back e.m.f. is equal to the supply voltage at time t_A . At time t_P the back e.m.f. is PQ and the voltage drop in the limiting resistance is QR.



The power dissipated in the resistance is thus $2I \cdot QR$, and the total energy lost in the starting process is therefore $2I$ times the area OBC.

Fig. 12 (b) shows the conditions in the second case. The motors speed up at the same rate as before and therefore the back e.m.f. of each motor is represented by the line OB, as before. The back e.m.f. of the series combination, however, is twice this value and is represented by OB', where N is mid-way between O and A and NM represents half the supply voltage. The voltage across the limiting resistance during the series period is thus QR, when the current is only I, so that the energy lost is I times the area OCB'. At time t_N the motors are switched into the parallel position and the back e.m.f. is represented by the line MB. The energy lost in this period of the starting is $2I$ times area MB'B, since the total current is now $2I$.

If we represent the supply voltage by V and the starting period by T , the energy lost in the first method is

$$2I \times OBC = 2I \times \frac{1}{2}V \times T = IVT.$$

In the series-parallel method the energy lost is

$$\begin{aligned} & I \times OB'C + 2I \times MB'B \\ &= (I \times \frac{1}{2}V \times \frac{1}{2}T) + 2I \times \frac{1}{2}(\frac{1}{2}V \times \frac{1}{2}T) \\ &= \frac{1}{2} IVT. \end{aligned}$$

The energy input to the motors in either method is

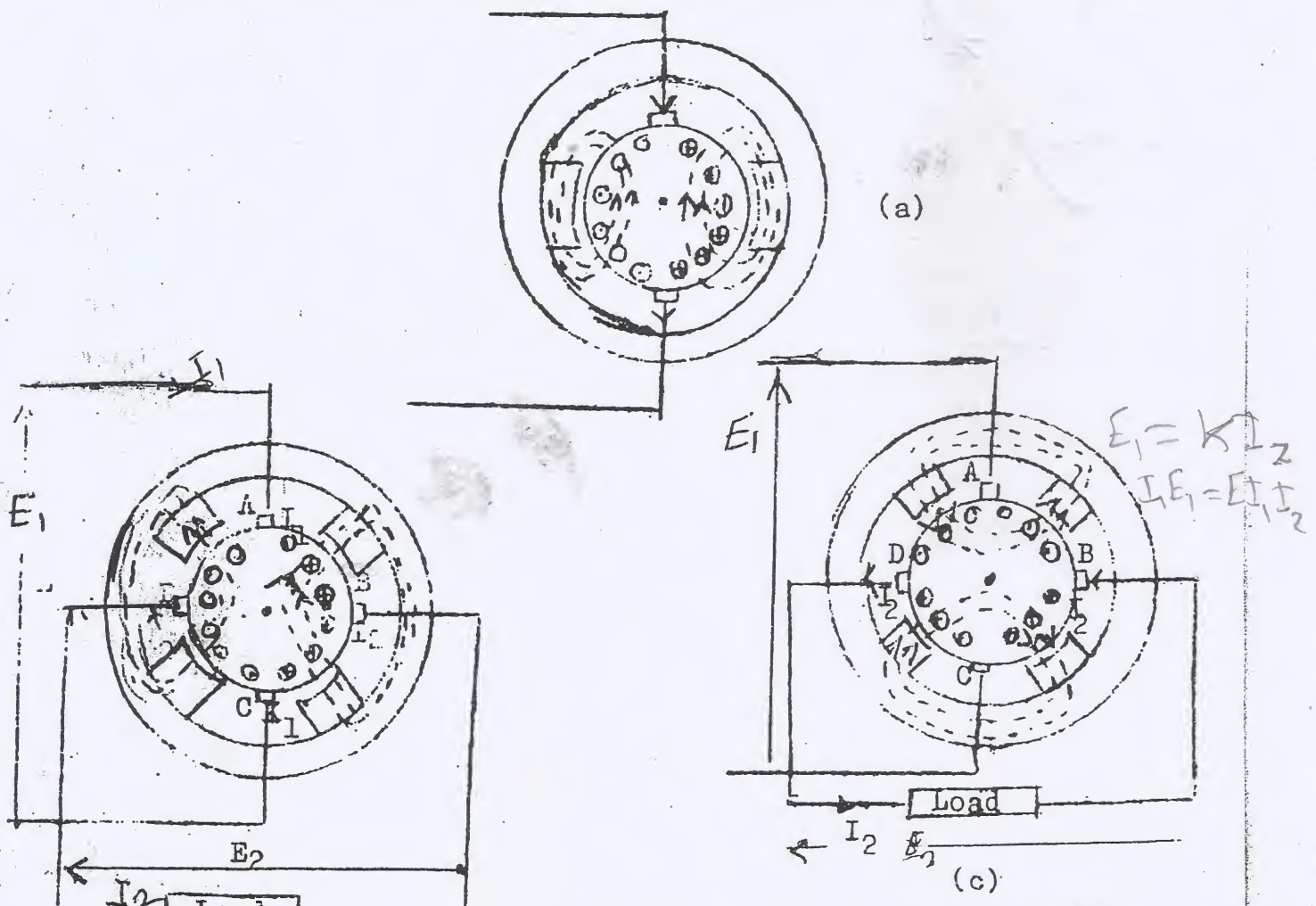
$$2I \times \frac{1}{2}V \times T = IVT.,$$

since each receives a current I at a mean voltage $\frac{1}{2}V$. Thus the efficiency of the first method is 50 per cent, whilst in the seriesparallel method it is $66\frac{2}{3}$ per cent. The series-parallel method enables a saving of 15 to 20 per cent of the total energy to be obtained in ordinary tramway running; moreover, it allows two running speeds (four if tap-field control is used in addition). If four motors are used they admit of series, series-parallel, and parallel combinations, and the losses in starting are 37.5 per cent of the energy used (as compared with 50 per cent in the series-parallel method and 100 per cent in the simple parallel method); there are three speeds (without the use of tap-field control) whose ratios are 1 : 2 : 4. This method is used in freight trains but not for trams, where series-parallel (most recently with tap-field control) is used.

In practice the effect of the resistance of the motor is not entirely negligible, but the method of calculating the performance of seriesparallel starting is not essentially

altered. The resistance drop in the motor is constant, since the current is considered constant, and this is subtracted from the supply voltage; the difference is available for back e.m.f. and voltage drop in the limiting resistance and the method is then as shown in Fig.12.

The Metadyne. In the methods of control described above, resistance is put in series with the motor and slowly cut out. During this process of notching, which is jerky, a great deal of energy is wasted in the resistances. The metadyne achieves smooth control without dissipating energy in a resistance. It is, in essence, a rotating transformer for d.c. power with a transformation ratio that can be varied (continuously, if desired). Thus it can draw power from a constant (d.c.) voltage source and deliver it at a constant current and varying voltage to an accelerating



motor; this is clearly the best way in which the motor can receive power.

The metadyne has a d.c. armature, but twice as many poles and brushes with the given armature as an ordinary d.c. machine. Fig.13 (a) shows an ordinary d.c. machine with two poles and two brushes: a current flowing in the direction shown causes the armature current distribution shown in the figure with the corresponding cross-flux, which is mainly restricted to the pole faces.

Fig.(b) shows the metadyne using the same armature. There are four poles and four brushes, as shown, and a current I_1 produces an armature current distribution as in Fig.13 (a). The flux due to the armature current is now provided with a path through the yoke by the four poles in the way shown. This primary flux produces an e.m.f. in the armature between the brushes B and D, so that a current I_2 flows through the load in the direction shown. The load current I_2 produces the armature-current distribution and flux shown in Fig.13 (c). This secondary flux produces an e.m.f. between brushes A and C, which neutralizes the applied voltage E_1 (except for the small resistance voltage-drops).

Suppose that the metadyne is run at a constant speed and that resistance voltage-drops are negligible. The e.m.f. produced between brushes A and C is E_1 and is due to the flux produced by current I_2 ; the flux due to I_1 produces e.m.f. between brushes B and D. We have therefore,

$$E_1 = KI_2$$

Similarly

$$E_2 = KI_1$$

where K is a constant depending on the construction of the machine and the speed. We see that

$$E_1 I_1 = E_2 I_2, \quad \dots \dots (9)$$

i.e. the input and output powers are equal. It is necessary, therefore, to supply only the running losses of the machine. Moreover if the supply voltage E_1 is constant, the load current I_2 is constant, no matter what the resistance of the load may be. If the load resistance increases, the load current remains fixed, but the input current increases to supply the necessary power.

The scheme shown in Fig.13 (b) is perfectly adequate to start a motor at constant current; the load is then merely the motor. Since the action is reversible (i.e. the currents can be reversed), this scheme would also give a system of regenerative braking, in which the motor sends back a constant I_2 to the set and thence I_1 to the line.

When the motor load has reached its maximum speed it is necessary to diminish I_2 to the running value. This is done by means of variator and regulator windings in the following way.

The variator winding is wound round the poles so that the flux lines are like those due to the secondary current, i.e. as shown in Fig.13 (c). The variator excitation is said to be positive if its flux is in the same direction as those due to I_2 , and negative if the flux is in the opposite direction. The use of the variator windings destroys the transformer property of the metadyne, as expressed in equation (9). For if the variator winding were given enough

C
V
S

current to produce the flux yielding the back e.m.f. E_1 , the current I_2 would fall to zero, and then there would be an input but no output. The metadyne would then speed up. Conversely if the variator flux were equal to the flux produced by I_2 in the absence of the variator winding and opposed it, I_2 would have to increase by 100 per cent to overcome this flux. We should then have an output equal to twice the input, and the metadyne would need mechanical power, equal to its input electrical power, to keep it running.

The metadyne is maintained as a transformer by means of a regulator winding, which produces a flux as in Fig.13 (b). This flux affects the output current and power, and if the current in the regulator winding is correctly adjusted, the output power remains equal to the input power.

The effects of the armature, variator, and regulator currents can be summarized in the form of the equations

$$\begin{aligned} E_1 &= KI_2 + k_v I_v \\ \text{and} \quad E_2 &= KI_1 + k_r I_r \end{aligned} \quad \left. \begin{array}{l}) \\) \end{array} \right\} \dots \dots \dots (10)$$

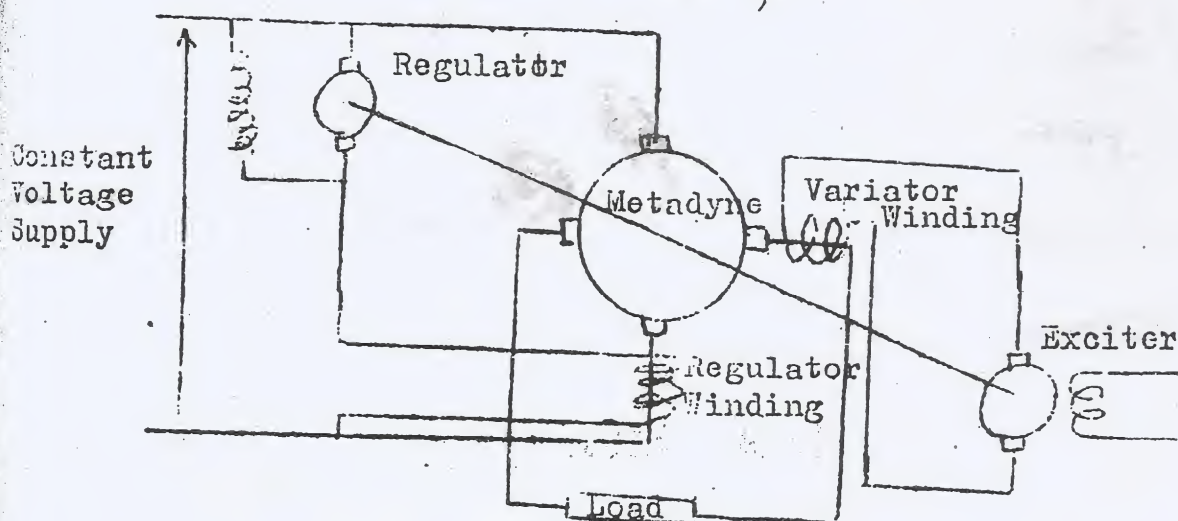


Fig.14 Complete Metadyne Set

where I_v and I_r are the variator and regulator currents, and k_v and k_r are constants of the machine, the windings, and the speed. The input and output powers are—

$$\begin{aligned} P_i &= E_1 I_1 = K I_1 I_2 + k_v I_1 I_v \\ \text{and } P_o &= E_2 I_2 = K I_1 I_2 + k_r I_2 I_r \end{aligned} \quad \left. \vphantom{\begin{aligned} P_i &= E_1 I_1 \\ P_o &= E_2 I_2 \end{aligned}} \right\} \dots \dots (11)$$

The condition for transformer action is

$$k_v I_1 I_v = k_r I_2 I_r \quad \dots \dots \dots (12)$$

Fig.14 shows a complete metadyne set, which may be used for motor starting and regenerative braking. The method of exciting the field of the exciter is determined by the required current-voltage curve of the load (i.e. secondary). Fig.15 shows a set of characteristics. The secondary current-secondary voltage curve is determined by the requirements of the load. The primary current curve is calculated from the fact that the input power equals the output power, i.e. $I_1 = I_2 E_2 / E_1$, E_1 in this case being 600 volts and E_2 is along the abscissa. It is assumed that in the set a secondary current of 200 amperes is required to produce a back e.m.f. of 600 volts in the primary circuit. From this assumption and the curve of secondary current, the curve of variator current is drawn for a variator winding having the same number of turns per pole as the armature winding (i.e. for $k_v = K$). Then from these curves and equation (12) the regulator current is calculated and the curve drawn, again with $k_r = K$. Modifications are required, of course, to allow for resistance drop, iron saturation, windage losses, etc.

The metadyne has applications wherever control of d.c. motors is required. The control is smooth and requires no switching, so that switchgear and arcing are avoided. In some cases it is cheaper than the Ward-Leonard system in

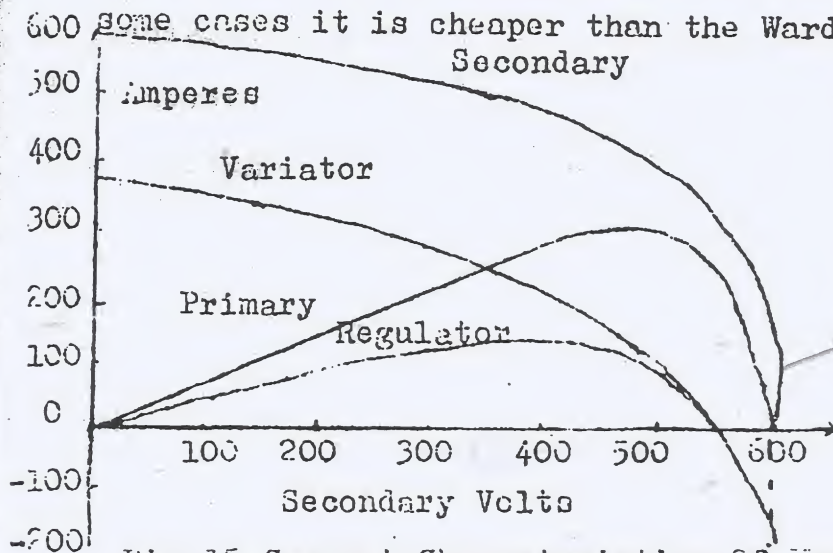


Fig.15 Current-Characteristics Of Metadyne Set

first cost. In traction it provides smooth acceleration, without skill on the part of the driver, and regenerative braking down to very low speeds. It is already being used on the Underground railway.

Field Weakening or Tapped Field Control. When the motors have run up to full speed, an increase of speed is still possible by cutting out some of the field turns by means of tapping or by a shunt. It is usual to have not more than two tappings giving 15 and 30 per cent increase in speed. The results are best explained by an example. $E = K\phi N$

EXAMPLE. The following figures relate to the series-wound motors of an electric locomotive- $E - IR = K\phi N$

Current per motor (amperes)	200	300	400	500
Train speed (m.p.h.)	41.5	33.5	28.5	28.0
Tractive effort per motor (lb.)	1 300	2 460	3 660	4 870

$$N = \frac{E - IR}{K\phi}$$

Calculate the values of speed and tractive effort for the same range armature current when the series field current is reduced 20 percent by a field diverting resistor.

A clear picture of the action of a field diverter is obtained by plotting speed and tractive effort against armature current: Fig.16

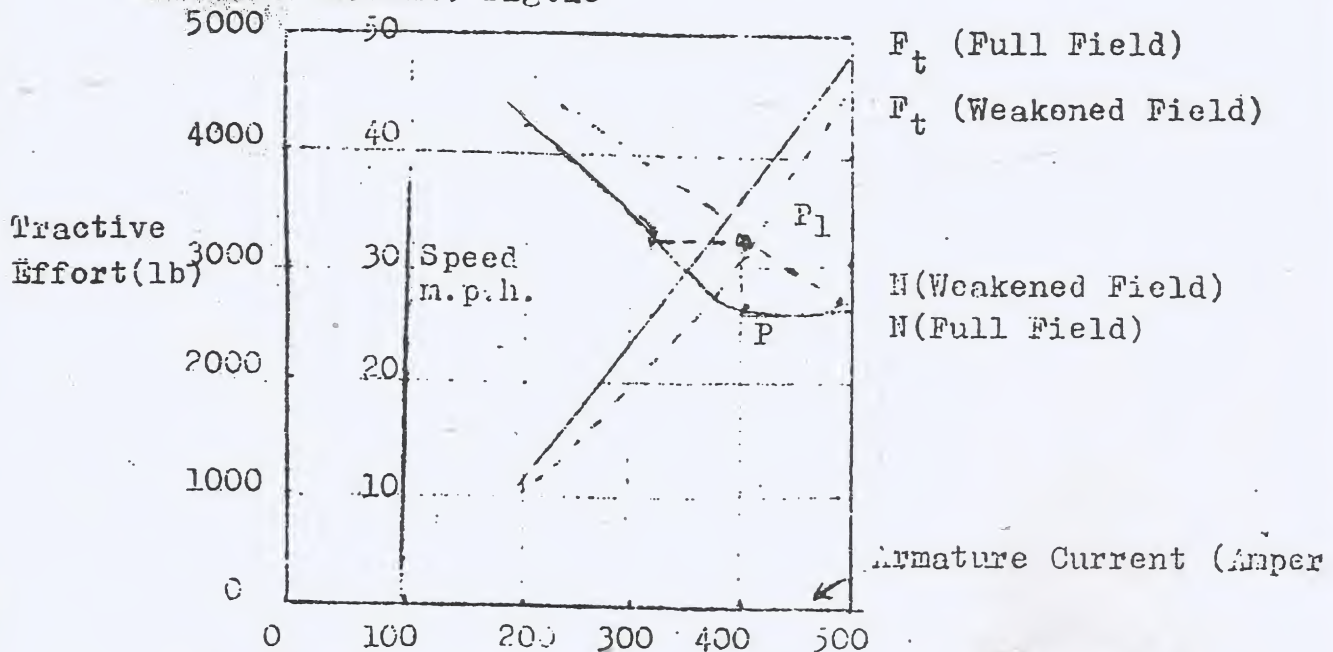


Fig.16 Field-Weakening Calculation

shows the curves for the values given above in full line. If the field current is reduced 20 per cent by a shunt resistance, the field current is 0.8 of the armature current. Thus when the armature current is 400 A., the flux is that produced by a normal field coil with $0.8 \times 400 = 320$ A., and as we ignore the resistance drop in the armature and assume that the supply voltage is constant, being equal to $N \phi$, the speed with the reduced field is that which occurred previously at 320 A. Thus the point P on the speed-current curve becomes the point P_1 . In this way we get the speed-current curve for the weakened field. The tractive effort

is proportional to $I\phi$, i.e. to I/N at constant voltage supply. The tractive effort at 400 A. with the weakened field is thus

$$3\ 660 \times \frac{28.5}{32} = 3\ 260\ \text{lb.}$$

Fig. 15 shows the new tractive effort-current curve. It is simple to derive the new tractive effort-speed from the two new curves.

Starting and Speed Control of A.C. Motors. The methods adopted differ considerably according as to whether the motor is three-phase or single-phase.

Three-phase Motors. Starting is done by means of liquid or metallic rheostats in the rotor circuit.

Speed control is effected in two ways, by cascading and pole changing. The effects of these methods are seen from

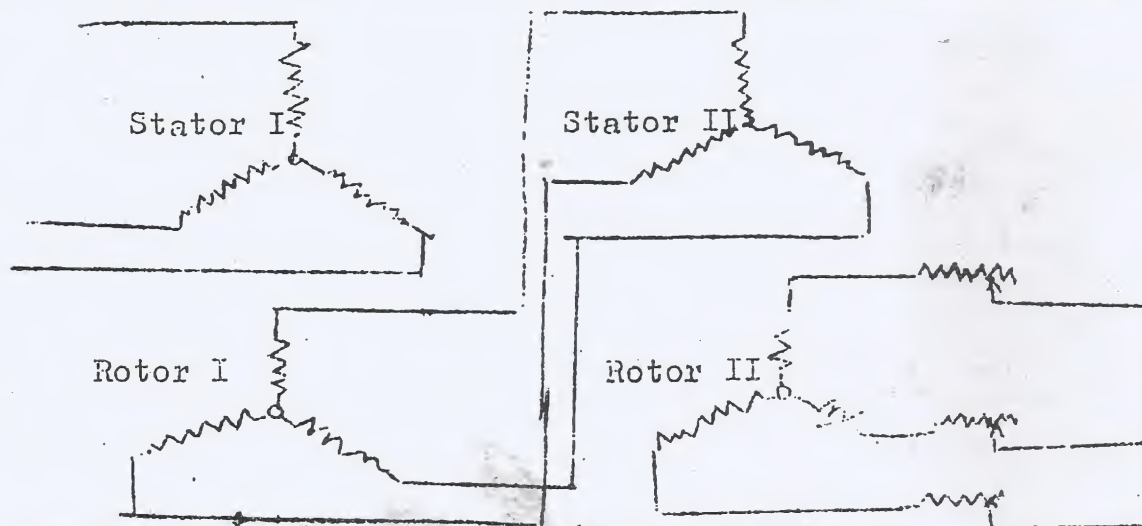


Fig. 17 Cascade Connection Of Motors (Electric Traction (Dover))

the following equations, which hold for an induction motor.

$$\text{Speed} = f(1 - s)/p \quad \dots \quad (13a)$$

$$\text{and Rotor power dissipated} = ksT/p, \quad \dots \quad (13b)$$

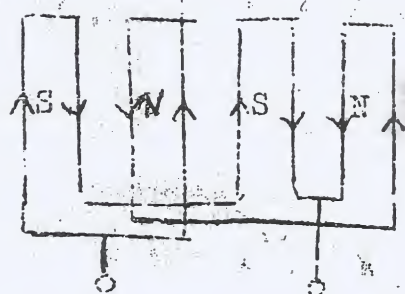
where f is the frequency, s the slip, p the number of pairs

$$s = \frac{N_s - N}{N_s} = 1 - \frac{N}{N_s}$$

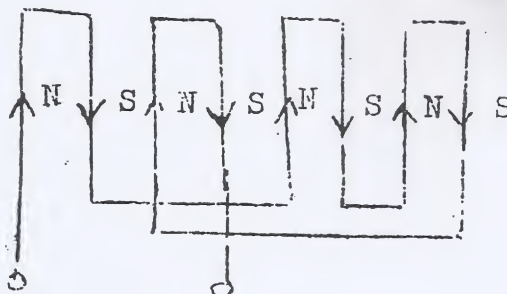
of poles, k a constant, and T the torque.

If power were dissipated in the rotor by means of resistance, the slip would increase by equation (13b) and hence the speed fall by equation (13a). Instead of wasting this power it may be used to drive another induction motor; and thus we achieve the cascade connection of Fig.17, in which power is taken by slip rings from the rotor of the first motor to drive the second motor. The rheostat in the second rotor enables speed regulation up to the cascade synchronous speed, which is $f/(p_1 + p_2)$, where p_1 and p_2 are the pairs of poles in the two machines, which are coupled mechanically. If the motors have equal numbers of poles, the cascade synchronous speed is half that of a single motor. The two motors provide approximately equal mechanical power. A disadvantage is the low power factor of the combination.

Equation (13a) shows that if the number of poles is changed the speed is changed. The ways in which the number of poles can be changed are numerous and complicated; but the principle may be illustrated simply as in Fig.18, where a winding is shown as giving 4 poles and 8 poles by altering the supply connections.



4- pole Winding

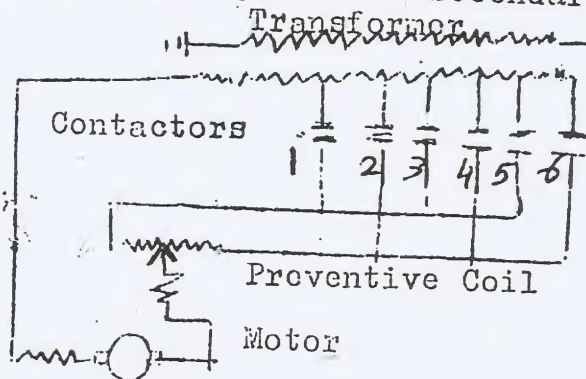


8- pole Winding

Fig.18 Pole Changing

Single-phase Motors. The voltage can be reduced on starting without the use of resistances, and this gives a large saving of energy; it should be noted, however, that this advantage of the a.c. system has been to a large extent neutralized by the introduction of the metadyne.

In the a.c. system, as the power is supplied from the line by a transformer, all that is necessary is a number ofappings on the secondary of this transformer.



Sequence of Contactors

Notch	Contactors					
	1	2	3	4	5	6
1	x	x				
2		x	x			
3			x	x		
4				x	x	
5					x	x

Fig.19 Connections For Contactor Method Of Tap-Changing (Electric Traction (Dover).

A preventive coil is used to ensure satisfactory operation, in a manner shown in Fig.19. In the case shown there are five notches, and at each position two adjacent contactors are closed. The preventive coil ensures that the part of the transformer secondary between the two contactors is not shorted. A very important advantage of this method is that each notch is a running position, so that there are available many speeds of running.

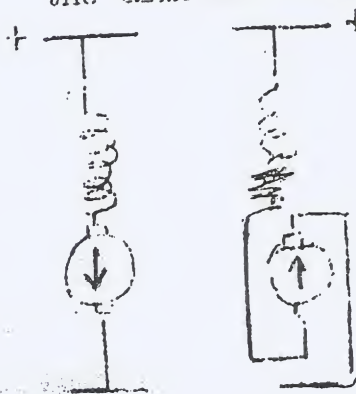
Electric Braking. On trains, trams, and trolley buses there are available mechanical and electrical brakes. The wheel brakes, which are mechanical, are worked by compressed air on trains and by hand on trams or trolley buses. On trams the mechanical track brake consists of one or more

Handwritten signature

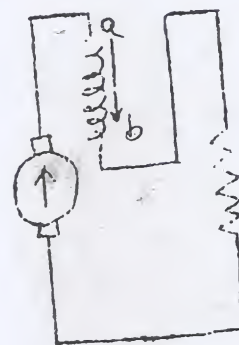
pairs of wooden blocks, which are pressed on to the track rails by means of levers; this brake is for use on steep gradients and utilizes the weight of the car. The magnetic track brake consists of electromagnets, which are suspended normally to clear the track; when they are energized they are attracted to the track.

There are three important methods wherein the kinetic energy of the tram or train is absorbed by an electrical process; these are (1) plugging, (2) rheostatic braking and (3) regenerative braking.

(1) In plugging, the torque of the motor is reversed and this brings the car to a standstill. In a d.c. motor a reversed torque is obtained by reversing either the field or the armature current (not both); it is usually convenient to



Running Plugging
Fig. 20 Plugging A Series Motor



Running Rheostatic Braking
Fig. 21 Rheostatic Braking With A Series Motor

reverse the armature current. Fig. 20 shows the running and plugging connections for a series motor. In the running position the back e.m.f. is nearly equal to the supply voltage and opposes it, so that a small voltage is available to drive the normal current through the small resistance of the motor. In the plugging position the back

e.m.f. is in the same direction as the supply voltage; so that at the instant of switching twice supply voltage is available and an enormous rush of current would take place (about twice the current taken by the stationary motor on full voltage). Limiting resistance has therefore to be inserted in series with the motor. During the braking period the supply has to give energy (at the rate of VI watts), and this energy plus the kinetic energy of the car has to be dissipated in the series limiting resistances. The method is thus wasteful of energy, although it is efficient for braking purposes.

Plugging can be achieved in an induction motor by reversing the direction of rotation of the magnetic field, and this is easily done by reversing the connections to two of the three phases. In this case the current does not increase to an excessive value. By using different values of rotor resistance, any desired speed-torque braking curve can be obtained.

(2) In rheostatic braking the motor is disconnected from the supply and connected to a resistance. The kinetic energy of the car drives the motor which then acts as a generator and dissipates energy in the resistance. This method can be used for d.c. and synchronous motors.

In the case of d.c. shunt and synchronous motors the field is kept across the supply, but the armature is switched from the supply to across a resistance; if the supply fails, the field disappears and there is no braking.

result that equal excitation is achieved in both machines.

Rheostatic braking cannot be used with induction motors.

(3) Plugging and rheostatic braking involve the wasting of the kinetic energy of the tram or train, whilst the former even draws more wasted energy from the supply during the braking period. A worth-while economy is effected if the kinetic energy of the vehicle can be turned into electrical energy and pushed back into the supply. This method is known as regenerative braking.

The induction motor acts automatically as a regenerative brake at speeds above the synchronous speed, and is of special advantage on mountain railways. It is found that the motor returns up to 20 per cent of the total energy on certain railway runs, and saves a great deal of brake shoe wear.

The series d.c. motor cannot be used for regenerative braking without modification. For if the motor is to act as a generator its armature current reverses and the series field connections must be reversed, otherwise the field flux will be neutralized and the build-up will not occur. But even if the driver were skilful enough to reverse the field connections at the exact moment, the method would still be useless. For at the instant of reversal the e.m.f. generated by the motor is small and is completely overpowered by the supply voltage, which drives current through the field in the wrong direction, reverses the field and causes the e.m.f. of the motor to aid the supply voltage. The result

is a short-circuit of the supply. The main trouble associated with regenerative braking by series motors is seen to be due to the lack of control of the field. There are various methods of overcoming this difficulty, either by modification of the windings or by supplying the machine with separate excitation.

Disadvantages and Advantages of Regenerative Braking on Level Routes :

In practice a number of difficulties and disadvantages are involved in the application of regenerative braking to level routes. The disadvantages, as far as d.c. equipments are concerned, are briefly.

① The motors are larger, heavier, and more costly than those for ordinary equipments, thereby resulting in more costly mechanical parts (e.g. trucks), an increase in the weight of the train, and possibly an increase in the number of motors.

② Additional equipment is necessary for the purpose of controlling and safeguarding the regenerative action of the motors and to obtain suitable regenerative characteristics. These features result in increased first cost of the trains, increased

③ maintenance charges on the electrical equipment, and increased complication in the control and method of operation. Moreover, difficulties in the operation of the sub-stations may occur. Sometimes the recuperated energy exceeds the energy output from the sub-station.

To offset the disadvantages there are the following advantages :

Reduced energy consumption : reduced wear of brake shoes and wheel tyres; lower maintenance costs for these items;

relatively small amount of brake dust produced when the mechanical brakes are applied.

Experience with regenerative equipments on tramway and trolleybus routes has shown that on level routes the energy consumption is about 10 per cent lower than that of a standard (series motor) equipment, the operating conditions being similar in each case. With undulating routes the saving may be of the order of 20 per cent. *87*



Fig. 24 : French Method of Regenerative Braking .

Electric Regenerative Possibilities on Main-line and Mountain

Railways : The operating conditions on main-line railways having long gradients and on mountain railways are very favourable to electric regenerative braking owing to (i) the relatively large amount of energy available during the descent of the gradients, (ii) the exclusive use of electric locomotives, (iii) the operating conditions permitting the use (when desirable) of motors having constant speed characteristics. In these cases, even when d.c. series motors are employed, the additional equipment necessary for regenerative braking adds

but a small percentage to the cost of the locomotive.

The advantages due to regenerative braking on these railways are greater than those obtained on level routes. Thus,, in addition to the saving of energy, there are large savings in the maintenance of the mechanical brakes and wheel tyres. Moreover, owing to the mechanical brakes being used only to a small extent - and, in some cases , not all all--during the descent of gradients, the danger of overheating of the brake shoes and wheel tyres (which may be a serious menace with mechanical brakes) is eliminated, thereby conducting to greater safety in operation and more uniform braking. Further, higher operating speeds on the gradients become possible and heavier trains can be taken down the gradients.

In these circumstances regenerative braking results in a considerable reduction in the operating costs compared with mechanical braking.

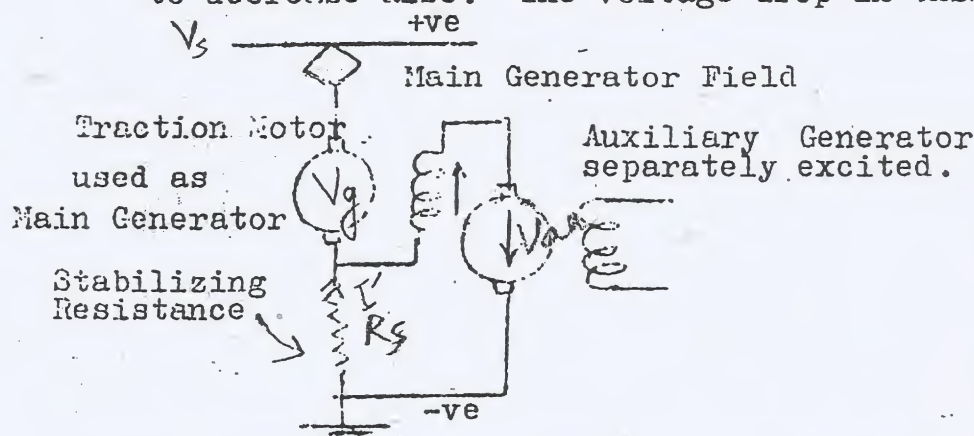
Practical Results. The Giovi-Genoa lines of the Italian State Railways form a striking example of the advantages of electric regenerative braking on a railway with heavy gradients. With electric traction the capacity of the lines has been trebled, due to the heavier trains which can be run on the down gradients and the higher speeds permissible with electric braking. The running costs have been found to be only about 75 per cent of those when the lines were operated with steam locomotives, although the plant of the generating station is not fully utilized. These low costs are the result

of electric recuperation of energy on the down gradients, the recuperated energy being of the order of from 60 per cent to 80 per cent of the energy consumption for the up journey with the same train. Considerable saving is also effected in the brake shoes, wheel tyres, and rails, as the mechanical brakes are only used for "slow-downs" and stops.

Fig.24 shows a well-known French method. During motoring the machine acts as a series motor, but has a main series field winding and auxiliary windings in parallel with it. During generation the auxiliary windings are switched (in series) across the supply, and the machine acts as a shunt generator slightly and differentially compounded. If there are several motors, there need not be any auxiliary windings. During motoring the field windings are in series with their respective armatures, and the motor circuits are in parallel. But during regeneration the circuit is as shown in the right-hand side of Fig.24 except that in place of a single armature we have all the armatures in parallel, and what are shown as auxiliary field windings are the ordinary series windings of all the motors but one.

Fig.25 shows the Metropolitan-Vickers regenerative system, which uses an auxiliary generator; this can be either one of the train motors or a special machine. The magnitude of the regenerated current is controlled by varying the field strength of the auxiliary generator, and thus the regeneration does not depend wholly upon the speed of the tram. The stabilizing

resistance is used to prevent current surges when the train crosses from one section of the supply to another, and to compensate for variable line voltage (towards which regeneration is very critical). Suppose that the line voltage rises, so that the regenerated current tends to decrease. The current in the stabilizing resistance, being the sum of the auxiliary generator and regenerated currents, tends to decrease also. The voltage drop in this resistance



$$V_g = V_s + I' R_s$$

$$I' R_s = V_g - V_s$$

$$I' = I + I_{au}$$

$$V_{au} = V_f + I R_s$$

If $I R_s$ decreases
 V_f increases
 V_g increase

Fig. 25 Metropolitan-Vickers Regenerative System

decreases and thus the voltage across the main generator field increases, so that the e.m.f. in the main generator increases and thus compensates for the rise of the line voltage.

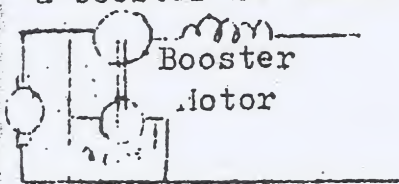
The modern tendency is to use regenerative braking down to about 10 m.p.h., then rheostatic braking down to 4 m.p.h., and finally mechanical braking to a standstill. This diminishes wear on the brake shoes.

Boosters. These are generators inserted into a circuit to compensate for a variable voltage drop. For instance if the current in a feeder varies, the voltage supplied to the

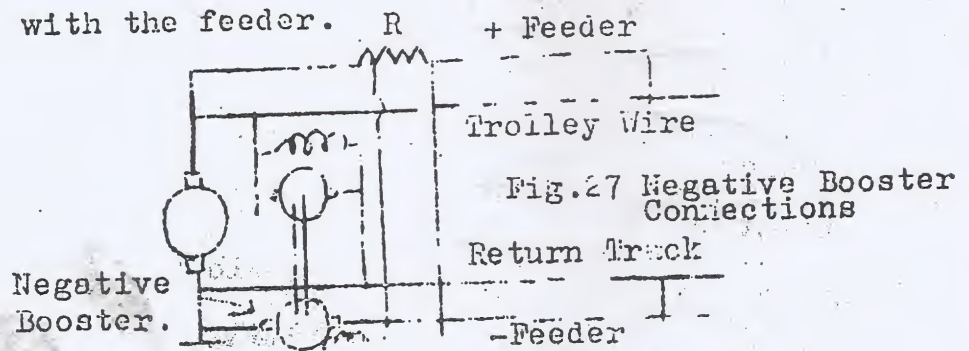
distributors may vary more than the legal amount. This difficulty may be overcome by using very large gauge feeders, but this is costly. A more economical method is to insert a feeder booster in the feeder. This booster is a series generator in which the e.m.f. is proportional to the field current, which is here the feeder current. By proper choice of the constants of the booster, the e.m.f. can exactly neutralize the voltage drop in the feeder. Fig. 26 shows the method adopted in practice. The booster is clearly a low-voltage, heavy current machine.

The effect of voltage drop in the feeders can be overcome by using compound d.c. generators, but the use of boosters is more convenient when there are feeders of different lengths.

In a tramway system it may be desirable to raise the voltage of the line at a distant point. This can be achieved by running a feeder from the generator to the point and inserting a booster in series with the feeder. $R + \text{Feeder}$



46.26 Feeder Booster



Negative
Booster.

NEGATIVE BOOSTERS subtract from the voltage, and are used in earth return systems in order to keep the potential of all points of the return rail within the Board of Trade regulation limit of 4.2 volts (to avoid the troubles of electrolysis).

Fig.27 shows how the negative booster is used; in this case a known fraction of the feeder current, which is shunted by

R, is used for the field winding.

Feeding and Distributing System for Tramways. The regulations demand that the voltage of the trolley wire shall not exceed 550 volts and the generating voltage 650; the potential difference between any two points of the earthed return must be less than 7 volts. and the potential of any point must not be more than 4.2 volts above earth. These conditions separate feeding systems for the trolley wire and the track rails.

Fig. 23 shows how the regulations are obeyed on a long system by the use of boosters. The negative bus-bar is earthed by two buried plates. One lead from the negative bus-bar is run to the track near the generating station, and one from the positive bus-bar to the trolley wire. If no other connections were used, the potential along the trolley wire would decrease with the distance away from the station; whereas the potential of the track would rise (because of the voltage drop of the current in it), so that a point would be reached where the potential would be greater than 4.2 volts. The potential along the trolley wire is kept constant within narrow limits by feeding the sections, which are isolated from each other, by feeders in series with positive or feeder boosters; in Fig. 23 one such booster is shown feeding the right-hand end of the trolley wire. The booster voltage regulates itself by the current in the feeder in series with the booster. The potential at the corresponding point of the track is lowered to earth potential by the negative booster, which is regulated by the

Ger

same feeder current. The positive and negative boosts are thus regulated by the load, and are such that the track potential is very low and the trolley potential is nearly constant (at about 550 volts). By supplying the trolley wire and track at sufficiently close intervals, the trolley wire can be kept as near 550 volts as desired and the track

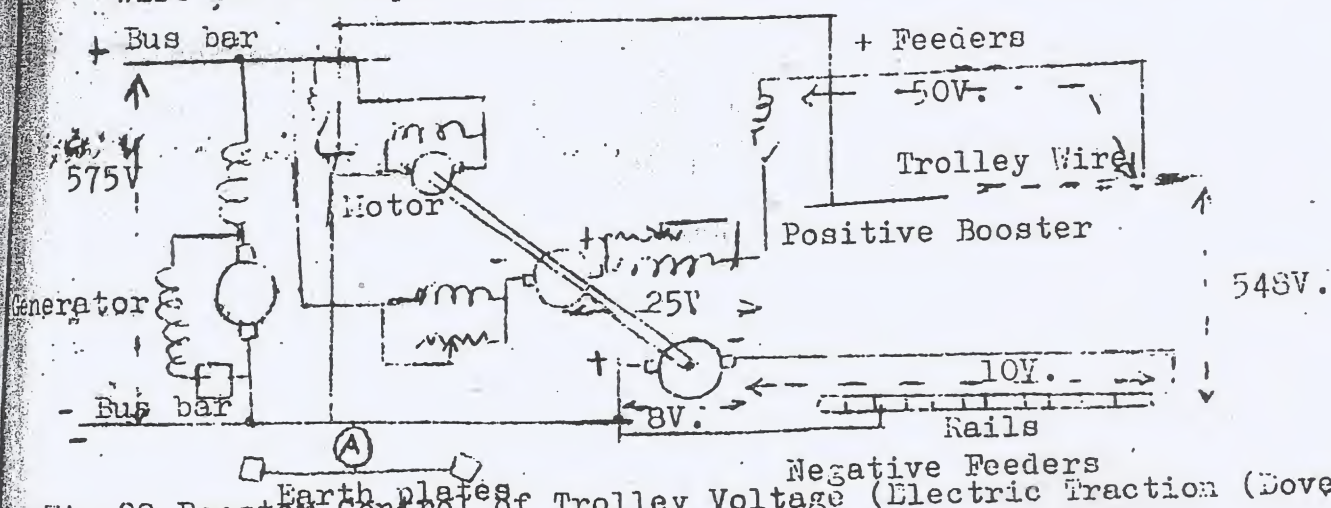


Fig. 28 Booster Control of Trolley Voltage (Electric Traction (Dover) is kept at nearly earth potential.

EXAMPLE. Explain with connection diagrams the function of (a) feeder boosters, (b) negative boosters, in an electric tramway system. A section of a tramway track 3 miles long has a resistance of 0.0145Ω per mile, and a uniformly distributed load of 320 A. per mile. A negative feeder having a conductor resistance of 0.0465Ω per mile is connected to the track at a point 2 miles from the station, and a negative booster is included in the circuit. If the potential of the track is reduced to zero at the point of connection to the booster, calculate the rating of the booster required and the maximum potential of the rails above earth.

Fig.29 shows the diagram of connections of the feeder and negative boosters. The function of the feeder is to keep the trolley wire at constant potential, whilst that of the negative booster is to keep any point of the track within 4 V. of earth potential and any two points within 7 V. potential difference.

We assume that in this case there is no feeder booster, and the scheme is as shown in Fig.29. Fig.30 shows the conditions of current and voltage. In the last mile 320 A. enter the trolley wire and 320 A. come in from the track (point P). At a point distance x from the generating station a current of $320(3-x)$ goes along the trolley wire:

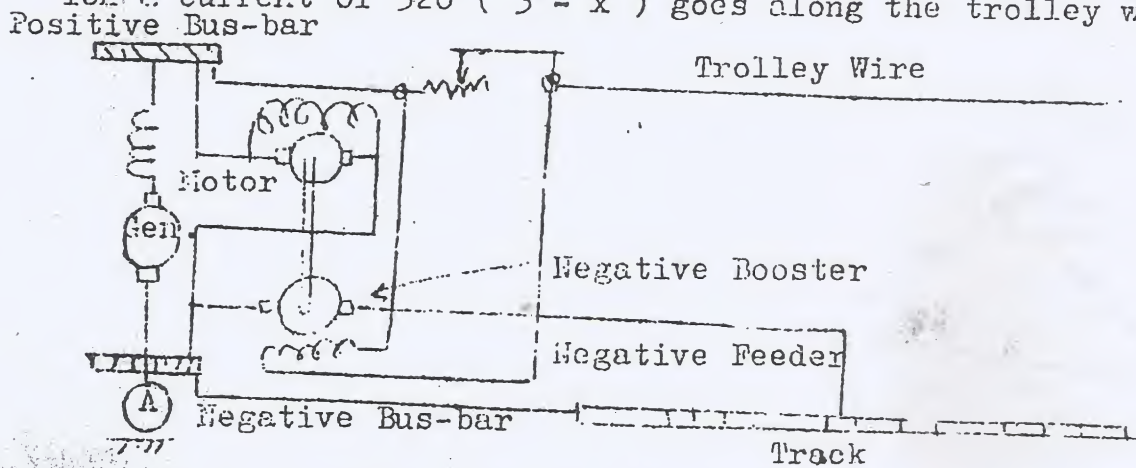


Fig.29 Negative Booster For Track

Let i_x be the current returning along the track, and let I be the current in the negative feeder. Then

$$i_x + I = 320(3-x) \text{ or } i_x = 320(3-x) - I.$$

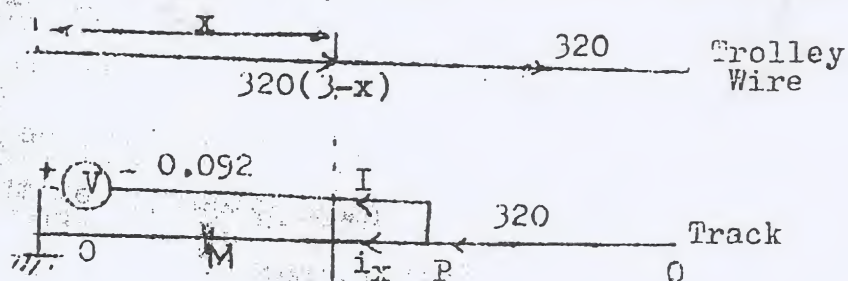


Fig.30

As the point P has been reduced to zero voltage, the voltage drop along OP is zero, i.e

$$\int_0^2 0.0145 i_x dx = 0.$$

Substituting for i_x we find

$$\begin{aligned} \int_0^2 [320(3 - x) - I] dx &= 0 \\ &= \left[320(3x - \frac{1}{2}x^2) - Ix \right]_0^2 \\ &= 1280 - 2I, \end{aligned}$$

i.e. $I = 640.$

The distribution of the current along the track is then the following. At M, one mile out, there is no current. Between M and O the current flows towards O, at which point the current is 320 A.; between M and P the current ~~flows towards~~ P, where it is 320 A. At Q the current is zero, and increases at P to 320 A. The maximum potential of the track occurs at M and Q, as current flows from higher to lower potentials. At these points the potential is

$$\begin{aligned} \int_0^1 (0.0145 \times 320x) dx \\ &= 0.0145 \times 320 \left(\frac{1}{2}x^2 \right) \Big|_0^1 \\ &= 0.0145 \times 320 \times \frac{1}{2} = 2.32 \text{ V.} \end{aligned}$$

If V is the voltage produced by the negative booster,

$$V - 0.0921I = 0,$$

giving $V = 59 \text{ V}$, The rating of the booster is

$$59 \times 640 \text{ VA.} = \underline{38} \text{ k VA.}$$

If the negative booster were not used, the maximum potential would occur at Q and be

$$\int_0^3 0.0145 \times 320 (3 - x) dx = 20.8 \text{ V.},$$

which is five times the permissible value.

The effect of the booster has been to reduce the voltage drop to that occurring in one-third of the track length, instead of that in the whole track length. It can be shown that if the track is of length l , resistance r per mile, and has a uniform load of i amperes per mile, the maximum potential is $\frac{1}{2}ril^2$. For the current at distance x from the station is $i(l-x)$, so that the maximum voltage, which occurs at the far end, is

$$\int_0^l ri(l-x)dx = ri \left[lx - \frac{1}{2}x^2 \right] = \frac{1}{2}ril^2.$$

Reducing the effective track length to one-third therefore reduces the maximum voltage to one-ninth. For example, in the case worked out above the maximum voltage is reduced from 20.8 to 2.32.

If the track is 5 miles long and negative feeders are run out to points at 2 and 4 miles from the station, the maximum voltage on the track is reduced to one twenty-fifth.

PROBLEMS ON ELECTRIC TRACTION

- 1) A train runs on a service in which there are two stops per mile and the schedule speed is 17 m.p.h. stops of 20 sec. duration. Determine the trapezoidal speed-time curve for the run if the acceleration is 1.2 m.p.h.s. and the braking retardation is 2 m.p.h.s. $V = 26.4$, $t_a = 22$ sec, $t_b = 13.2$ sec, $t_c = 50.6$ sec
- 2) A train is required to run between stations 1 mile apart at a schedule speed of 25 m.p.h., the duration of the stops being 20 sec. The braking retardation is 2.25 m.p.h.s. Assuming a trapezoidal speed time curve calculate the acceleration if the ratio (max. speed / average speed) is to be 1.25. $a = 1.1$ m.p.h.s
- 3) A train is accelerated uniformly from rest until a speed of 25 m.p.h. is reached 20 sec. after starting. Power is then cut off and the train coasts for 40 sec. The brakes are applied and the train is brought to rest 70 sec. after starting. The retardation during coasting may be assumed to be uniform at the rate of 0.1 m.p.h.s. Determine the distance run from start to stop and the average speed. $(354 \text{ m}, 18.2 \text{ m.p.h.})$
- 4) A train is required to run between stations 1.2 miles apart at a schedule speed of 25 m.p.h., the duration of the stops being 20 sec. The run is to be made according to quadrilateral speed - time curve and the coasting and braking retardations may be assumed at 0.1 m.p.h.s. and 2 m.p.h.s. respectively. Determine the acceleration if the speed at the end of the acceleration period is 38 m.p.h. Determine also duration of the coasting period.

$a = 1.257 \text{ m.p.h.s.}, t_c = 11.2$

- 5) A train is required to run between stations 1 mile apart at an average speed of 25 miles per hour. The run is to be a quadrilateral speed-time curve, the acceleration being 1.25 m.p.h.s. and the coasting and braking retardation being 0.1 and 2 m.p.h.s. respectively. Determine the duration of the accelerating, coasting, and braking periods and the distances run during these periods.

$t_a = 27.7, t_b = 11.7, t_c = 10.5 \text{ sec}, d_a = 128, d_b = 0.38, d_c = 0.84$

- 6) A train service between 2 stations 1 mile apart, and between which there is a uniform gradient of 1 in 80, is scheduled at an average speed of 25 m.p.h. in one direction to up the gradient, and 27.5 m.p.h. in the opposite direction. The dead weight of the train is 210 tons.

When operating on level track the acceleration is 1.2575 m.p.h.s. and the braking retardation is 2 m.p.h.s., the corresponding net tractive efforts being 30,000 lb and 47,000 lb. Calculate the specific energy output for runs in both directions made to trapezoidal speed time curves.

Assume the accelerating wt to be 10% greater than the dead weight and the train resistance is 12 lb/ton

$\frac{102.75}{635} \text{ watt hr / ton mile}$

- 7) A 250 ton electric train runs in main line service has an average of 32 m.p.h. between stations on the level situated 1.25 miles apart. The accelerations at starting is 1.25 m.p.h.s. and the braking retardation 2.3 m.p.h.s. Assuming a trapezoidal speed-time curve. Calculate the energy consumption for the run. Assume a train resistance has an average of 12 lb per ton allow 10% for the effect of rotational inertia

8) A D-C series traction motor has the following ch⁵.

Current (A)	100	2000	300	400
Speed (m.p.h.)	41	28	23.5	21.3
Tractive effort (lb)	650	1900	3400	5000

The ratio of the effective weight to the dead weight of the train is 1.1 to 1 and the braking retardation is 2m.p.h.s. A run of 1.5 mile is to be made in 110 sec. Draw the speed time curve knowing that the train resistance is 11.1lb/ton and the train has a total weight of 116 tons.

Give a diagram of connections and explain the action of a negative booster. A section ABC of an uninsulated rail return system is 3 miles long. A is earthed, and B is 2 miles from A. A negative feeder, with booster in circuit, is tapped to the rail at B. The loading is 400 A. per mile and may be assumed to be uniform.

Determine the maximum p.d. between any two points on the rail system, assuming no leakage, if the potential of B is 2.5 V. below earth. Determine also the output of the booster. The resistance of the rail system is 0.035 per mile. The resistance of the negative feeder is 0.03